1. (15 pts) Below are the graphs which correspond to the system of linear equations,
\[
\begin{align*}
-2x + 4y &= 8 \quad (eq_1) \\
3x - y &= -6 \quad (eq_2).
\end{align*}
\]
(a) Label the lines \(l_1\) and \(l_2\), where \(l_1\) corresponds to \(eq_1\) and \(l_2\) corresponds to \(eq_2\).
The line that is higher on the right hand side is \(l_2\).
(b) Estimate the point of intersection.
The point of intersections looks like \((-1.5, 1)\).
(c) Find the point of intersection algebraically.
Solve for \(y\) in the second equation by adding \(y\) and 6 to both sides:
\[3x + 6 = y.\]
Substitute this into the first equation and solve for \(x\):
\[-2x + 4(3x + 6) = 8 \rightarrow -2x + 12x + 24 = 8 \rightarrow 10x = -16 \rightarrow x = \frac{-16}{10} = -1.6\]
Now substitute this \(x\)-value into \(3x + 6 = y\) and solve for \(y\):
\[y = 3(-1.6) + 6 = -4.8 + 6 = 1.2.\]
So the solution to the system is \((x, y) = (-1.6, 1.2)\).
(d) Shade in the region that corresponds to
\[
\begin{align*}
-2x + 4y &\geq 8 \\
3x - y &\leq -6.
\end{align*}
\]
The region that is mostly in the second quadrant should be shaded in. A good test point would be \((-2, 6)\). Plug these numbers into the equalities and you’ll see that both are satisfied:
\[-2(-2) + 4(6) = 4 + 24 = 28 \geq 8 \]
\[3(-2) - (6) = -6 - 6 = -12 \leq -6.\]

2. (20 pts) Simplify the following expressions, show the exponent rules you use, and eliminate all negative exponents. Assume all variables represent positive numbers. DO NOT GIVE SOLUTIONS AS DECIMAL REPRESENTATIONS.
(a) \(-2^4 - (2)^4 = -2^4 - 2^4 = -2(2^4) = -2^5 = -32\)
(b) \(\frac{7^{21}}{7^{23}} = 7^{21-23} = 7^{-2} = \frac{1}{7^2} = \frac{1}{49}\)
(c) \(\sqrt{18} - \sqrt{8} = \sqrt{9\times 2} - \sqrt{4\times 2} = \sqrt{9\sqrt{2}} - \sqrt{4\sqrt{2}} = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}\)
(d) \((-xy^2)^{-2} \left(\frac{1}{2} x^2 y\right)^4 = \left(\frac{1}{-xy^2}\right)^2 \left(\frac{x^2 y}{2}\right)^4 = \left(\frac{1^2}{(-x)^2 y^4}\right) \left(\frac{x^8 y^4}{2^4}\right) = \left(\frac{1}{x^2 y^4}\right) \left(\frac{x^8 y^4}{16}\right) = \frac{x^8 y^4}{16x^2 y^4} = \frac{x^8 y^4}{16} = \frac{x^6}{16}\)
3. (20 pts) The following table gives the balance of a certificate of deposit $t$ months after it is purchased.

<table>
<thead>
<tr>
<th>month ($t$)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>balance ($B$)</td>
<td>5000</td>
<td>5040.63</td>
<td>5081.58</td>
<td>5122.87</td>
<td>5164.49</td>
<td>5206.45</td>
<td>5248.76</td>
</tr>
</tbody>
</table>

(a) From the table, determine the factor by which the balance grows each month.

If $B = f(tx) = c * a^t$, then $a = \frac{f(1)}{f(0)} = \frac{5040.63}{5000} = 1.00813$

(b) Find a function $B = f(t)$ which describes the growth of the above CD.

If $B = f(x) = c * a^x$, $a = 1.00813$ and $f(0) = c = 5000$, then $B = f(t) = 5000 * (1.00813)^t$.

(c) What is the estimated value of the CD after one year?

One year is $t = 12$ months, so $f(12) = 5000 * (1.00813)^{12} = 5000 * (1.10199) = $5509.95

(d) What is the annual percent growth rate (interest rate) of the CD?

Since the multiplying factor for one year is $a = 1 + r = 1.10199$, we have $r = 1.10199 - 1 = 0.10199$ and so the annual percent growth rate is 10.199%.

4. (10 pts) The radioactive isotope gallium-67 decays by 1.48% every hour.

(a) If there are initially 100 milligrams of gallium-67, find a formula for the amount remaining after $t$ hours.

$R = R_0(1-r)^t$ where $R_0 = 100$mg, and $r = 0.0148$. So $R = 100(0.9852)^t$.

(b) The half-life of gallium-67 is about 46.5 hours. What does this mean?

It will take 46.5 hours for the amount of gallium-67 to reach 50mg.

(c) After 262.4 hours there is only 2 milligrams of the original 100 milligrams of gallium-67. At what time will there only be 1 milligram left?

Since 1mg is half of 2mg, it will take an entire half-life. Thus, after $262.4 + 46.5 = 319.9$hours, the amount of gallium-67 will be 1mg.

5. (10 points) Suppose an exponential function, $y = f(x)$, is such that $f(-1) = 32$ and $f(1) = 2$.

(a) Do you expect the base of the exponential, $a$, to be greater than 1 or less than 1? Why?

Since $f$ decreases from $x = -1$ to $x = 1$, there is decay, so $0 < a < 1$. 
(b) **Find the equation for** \( f \).

6. (20 points) **Match the function to the graph (write the name of the function on the graph). If there is no function to match to a graph, write ”No Match”.

(a) Graph A: \( f(x) = 3 \cdot 1.5^x \)

(b) Graph B: \( h(x) = 3 \cdot \frac{1}{2}^x \)

(c) Graph C: \( g(x) = 3 \cdot 2.5^x \)

(d) Graph D: No Match

(e) Graph E: No Match

(f) Graph F: \( j(x) = 3 - 1.5x \)

(g) Graph G: \( p(x) = 3 + 1.5x \)

(h) Graph H: \( k(x) = 3 \cdot \left(\frac{1}{4}\right)^x \)