1. Find the 3rd degree Taylor polynomial for \( f(x) = \sqrt{x} \) about \( x = 4 \).

2. Knowing that \( e^u = \sum_{k=0}^{\infty} \frac{u^k}{k!} \) for all \( u \);
   
   (a) Find the sum of the series \( 1 - 1 + \frac{1}{2} - \frac{1}{6} + \frac{1}{24} - \frac{1}{120} + \cdots \).
   
   (b) Find a series representation for \( \int e^{-x^2} \, dx \).
   
   (c) Use an appropriate \( u \) and the MacLaurin series for \( e^u \) to find the smallest \( n \) so that the approximation \( \sqrt{e} \approx P_n(x) \) is accurate to within two decimal places.

3. (a) Show, using the definition, that the Taylor series for \( f(x) = \sin(\frac{x}{2}) \) about \( x = \pi \) is given by
   \[
   1 - \frac{(x - \pi)^2}{2^22!} + \frac{(x - \pi)^4}{2^44!} - \cdots \tag{1}
   \]
   (b) Write the above series (1) in closed form using summation notation.
   
   (c) Find the interval of convergence for the above series.
   
   (d) Using the \( n^{th} \) Taylor Remainder, show
   \[
   \sin(\frac{x}{2}) = 1 - \frac{(x - \pi)^2}{2^22!} + \frac{(x - \pi)^4}{2^44!} - \cdots
   \]
   for all \( x \) in the interval of convergence you found in (c).

   Note: \( \lim_{n \to \infty} \frac{a^n}{n!} = 0 \) regardless of the value of \( a \).