The Chain rule is used to differentiate a function that is the result of composing two or more “smaller” functions.

- Suppose \( y = f(g(x)) \), then \( \frac{dy}{dx} = f'(g(x)) \cdot g'(x) \). This requires that you be able to identify the functions \( f \) and \( g \).

1. Differentiate \( y = (2x^3 + x)^4 \)

   Here \( f(x) = x^4 \), \( g(x) = 2x^3 + x \), \( f'(x) = 4x^3 \) and \( g'(x) = 6x^2 + 1 \). For \( y' \), we need to compose \( f' \) with \( g \), that is \( f'(g(x)) = 4(2x^3 + x)^3 = 4(x^3 + x)^3 \). Finally we substitute the required pieces into the Chain Rule to get
   \[
   \frac{dy}{dx} = f'(g(x)) \cdot g'(x) = [4(2x^3 + x)^3] \cdot [6x^2 + 1] = 4(6x^2 + 1)(2x^3 + x)^3
   \]

2. Differentiate \( y = \sec \left( 4\pi x - \frac{\pi}{3} \right) \)

   Here \( f(x) = \sec x \), \( g(x) = 4\pi x - \frac{\pi}{3} \), \( f'(x) = \sec x \tan x \) and \( g'(x) = 4\pi \). For \( y' \), we need to compose \( f' \) with \( g \), that is
   \[
   f'(g(x)) = \sec(g(x)) \tan(g(x)) = \sec \left( 4\pi x - \frac{\pi}{3} \right) \tan \left( 4\pi x - \frac{\pi}{3} \right).
   \]
   Finally we substitute the required pieces into the Chain Rule to get
   \[
   \frac{dy}{dx} = f'(g(x)) \cdot g'(x) = \sec \left( 4\pi x - \frac{\pi}{3} \right) \tan \left( 4\pi x - \frac{\pi}{3} \right) \cdot [4\pi]
   = 4\pi \sec \left( 4\pi x - \frac{\pi}{3} \right) \tan \left( 4\pi x - \frac{\pi}{3} \right)
   \]

3. Differentiate \( y = \sin^2 \left( \frac{x}{5} \right) \)

   Here we have the composition of three functions, \( y = f(g(h(x))) \), and now
   \[
   \frac{dy}{dx} = f'(g(h(x))) \cdot g'(h(x)) \cdot h'(x)
   \]
   where \( f(x) = x^2 \), \( g(x) = \sin x \), \( h(x) = \frac{x}{5} \), \( f'(x) = 2x \), \( g'(x) = \cos x \) and \( h'(x) = \frac{1}{5} \). For \( y' \), we need to compose \( f' \) with \( g(h(x)) = \sin \left( \frac{x}{5} \right) \), that is \( f'(g(h(x))) = 2(\sin(h(x))) = 2\sin \left( \frac{x}{5} \right) \). We also need to compose \( g' \) with \( h(x) \), that is \( g'(h(x)) = \cos(h(x)) = \cos \left( \frac{x}{5} \right) \). Finally we substitute the required pieces into the Chain Rule to get
   \[
   \frac{dy}{dx} = f'(g(h(x))) \cdot g'(h(x)) \cdot h'(x) = \left[ 2\sin \left( \frac{x}{5} \right) \right] \cdot \left[ \cos \left( \frac{x}{5} \right) \right] \cdot \left[ \frac{1}{5} \right]
   = \frac{2}{5} \sin \left( \frac{x}{5} \right) \cos \left( \frac{x}{5} \right)
   \]