Optimal Positioning of Fixed Solar Panels

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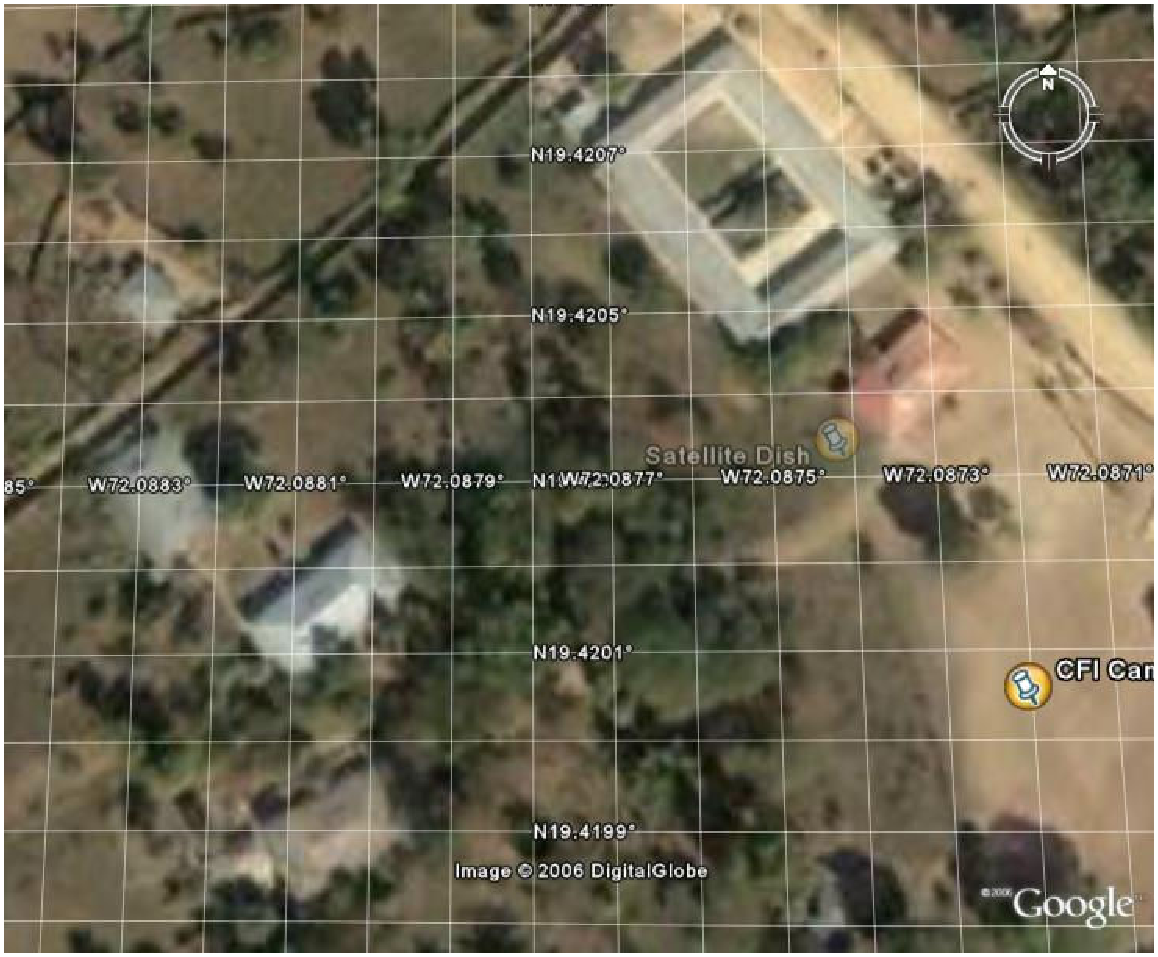
Background – Why Solar Panels?

- The problem arose on my sabbatical in Ranquitte, Haiti, in 2006-2007.
- My focus was on environmental problems, of which power delivery is one.
- We considered “tracking panels”; constraints of cost and simplicity meant that fixed panels – non-moveable – were the only alternative.
- It was a good project for my students. Math provides us with tools: if all we have is a hammer, everything looks like a nail; but if we don't know the hammer, we may not see the nails.

The CFI Campus, Ranquitte, Haiti: Five Solar Systems



Google Earth is a marvelous thing....

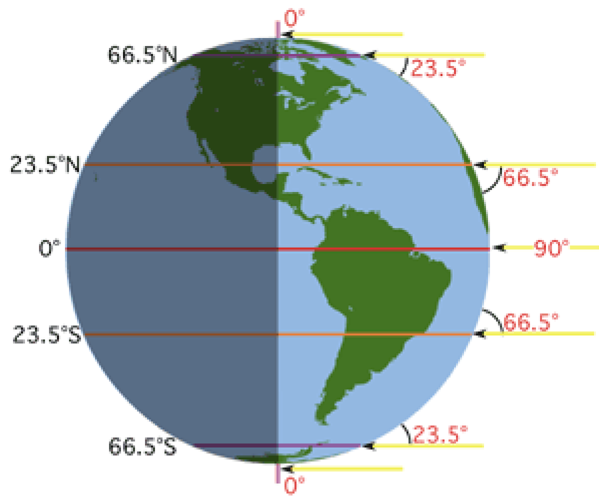
- School (for about 1000 students)
- Clinic
- Dorm (my office)
- Missionary's house (and ours!)
- Solar water pump

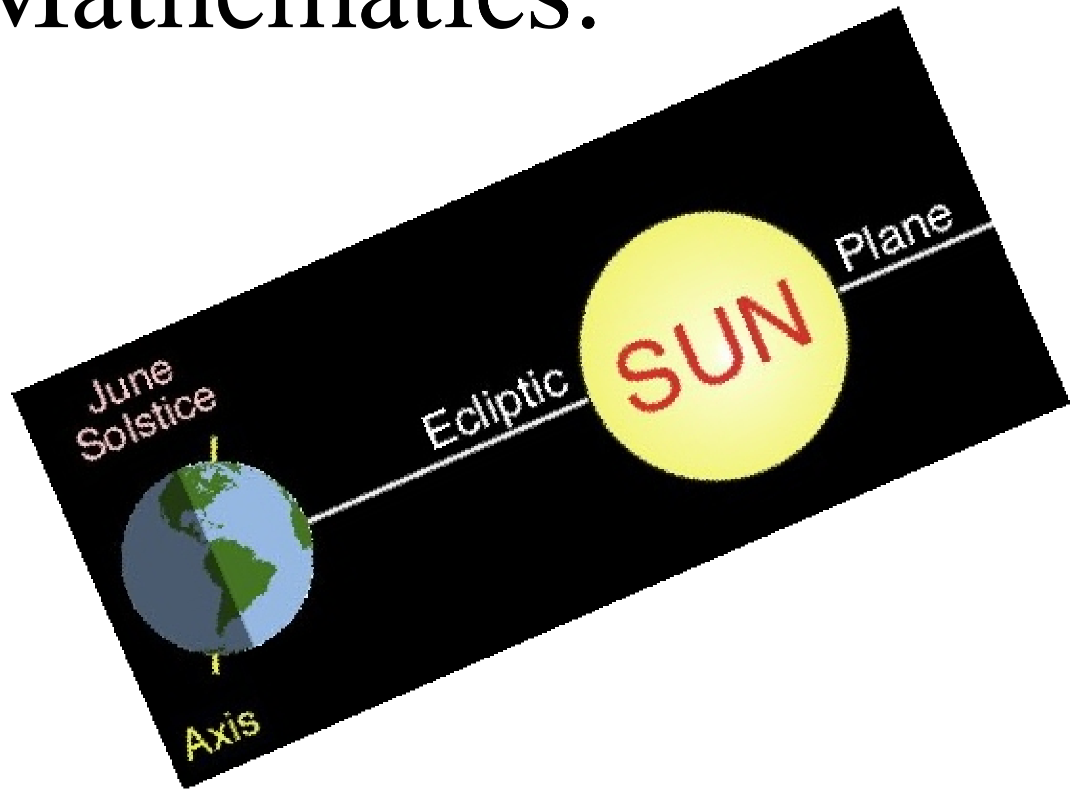
What Assumptions Were Made?

As we thought about how to calculate the best position of fixed solar panels, we realized that we would need to simplify the problem in order to get something tractable. In particular, we assumed the following (at least):

- Spherical Earth
- Circular Earth orbit about the Sun
- Flat horizons
  (bad for Haiti;
  good for calculus and linear algebra students)
- Atmospheric effects are negligible

Basics of the Mathematics:



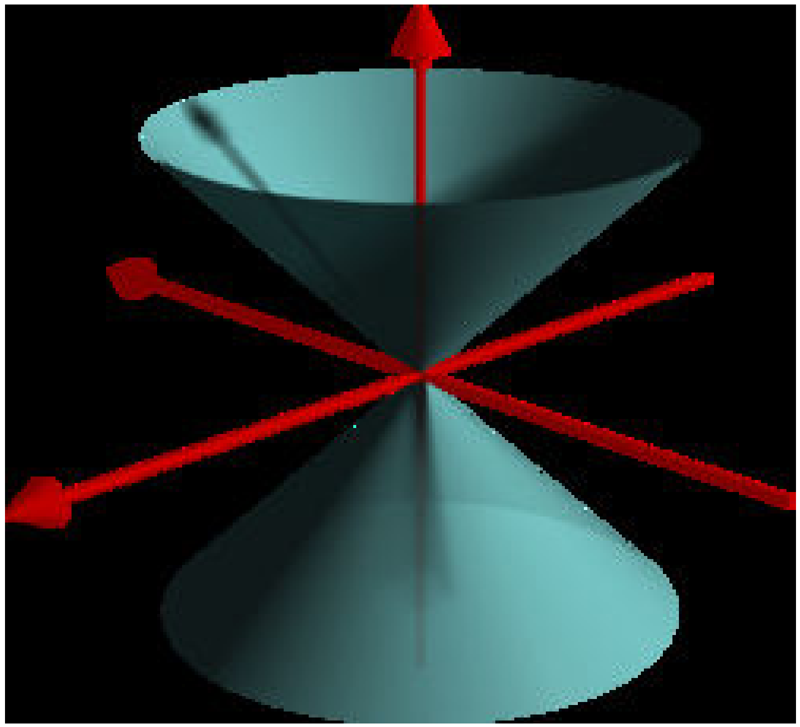


This is the basic setup: we'll take as our coordinate system the xy-plane of the equator, and the z-axis through the poles.

The sun is at the tip of a vector with zero y-coordinate that oscillates between the angles of the Tropics of Cancer and Capricorn.

The Earth rotates about the z-axis; the coordinates don't!

Let's do some math:



Your panel goes into shadow when time of day t and the time of year τ satisfy

$$\mathbf{c}(t) \cdot \mathbf{s}(\theta(\tau)) = 0$$

You:

$$\mathbf{c}(t) = \begin{bmatrix} \cos(lat)\sin(t) \\ \cos(lat)\cos(t) \\ \sin(lat) \end{bmatrix}$$

Sun:

$$\mathbf{s}(\theta(\tau)) = \begin{bmatrix} \cos(\theta(\tau)) \\ 0 \\ \sin(\theta(\tau)) \end{bmatrix}$$

where

$$\theta(\tau) = 23.4\sin(2\pi\tau)$$

is the angle that the Sun makes with the Equatorial plane at time of year τ

Solve for sunrise and sunset:

$\mathbf{c}(t) \cdot \mathbf{s}(\theta(\tau)) = 0$ gives rise to the following solutions:

$$t_1 = \text{asin}(-\tan(\theta(\tau))\tan(lat)) \quad \text{(sunrise)}$$
$$t_2 = \pi - t_1 \quad \text{(sunset)}$$

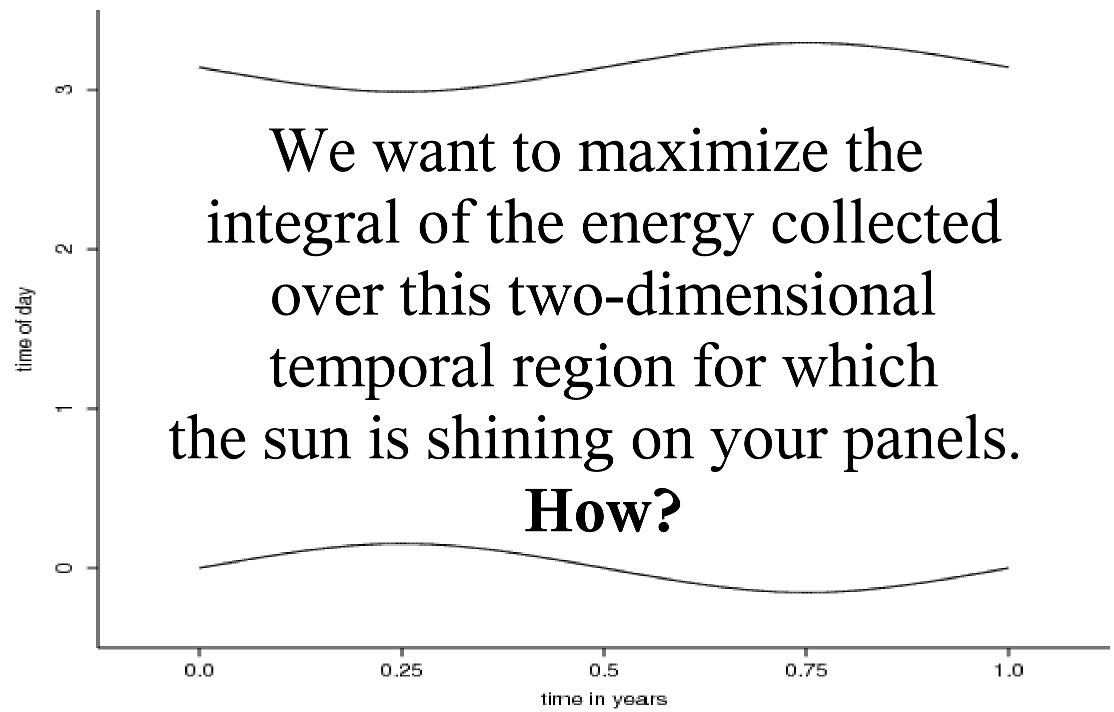
Now we know when the sun is shining. Our job is to collect the most sunlight over the time allotted throughout the year:

(This graph is a function of latitude.)

Day is 2π long.

t=0 is **6 a.m**.

τ – years



We want to maximize the integral of the energy collected over this two-dimensional temporal region for which the sun is shining on your panels.

How?

Energy collected depends on pitch of the panel (or the "pitch latitude", p):

$$TE_{lat}(lat) = \int_{-1/4}^{1/4} \int_{t_1(\theta(\tau),lat)}^{\pi/2} \mathbf{c}(t) \cdot \mathbf{s}(\theta(\tau)) \; dt \; d\tau$$

$\mathbf{c}(t)$ is an (implicit) function of lat. Here's the bait and switch:

$$TE_{lat}(p) = \begin{cases} \cos(p) \int_{-\frac{1}{4}}^{\frac{1}{4}} \cos(\theta(\tau)) \cos\left(t_1(\theta(\tau), lat)\right) \; d\tau \\ + \\ \sin(p) \int_{-\frac{1}{4}}^{\frac{1}{4}} \left(\frac{\pi}{2} - t_1(\theta(\tau), lat)\right) \sin(\theta(\tau)) \; d\tau \end{cases}$$

$$TE_{lat}(p) = \int_{-1/4}^{1/4} \int_{t_1(\theta(\tau),lat)}^{\pi/2} \left\{ \begin{array}{c} (\cos(\theta(\tau)) \cos(p) \sin(t) + \\ \sin(\theta(\tau)) \sin(p)) \end{array} \right\} \; dt \; d\tau$$

So it all boils down to this:

$$TE_{lat}(p) = \alpha \cos(p) + \beta \sin(p)$$

Optimization: we have a best pitch!

$$\frac{dTE_{lat}(p)}{dp} = \alpha(-\sin(p)) + \beta\cos(p)$$

Setting the derivative to zero gives

$$p(lat) = \operatorname{atan}\left(\frac{\beta}{\alpha}\right)$$

which yields a maximum, as

$$\frac{d^2TE_{lat}(p)}{dp^2} = -TE_{lat}(p) < 0$$

So pitch your panels so that they're pointing like **c**(t) at latitude p, and you'll have an optimal solution among fixed positions.

Next stop: **Linear algebra!** Given that the panels are on a pitched roof which doesn't face south, how should one place the panels on the roof to maximize collection?

But be careful what you ask....

(you might be expected to do it – maybe even twice!).

I've made the calculations a little bit easier for the next person, by creating a web page where the essential calculations are carried out:

For folks at WKU: what's the bang?

Latitude of Western Kentucky University (35.97 Degrees):

$TE_{35.97}$	Panel Position
0.397098	Flat, pointing straight up
0.468875	Best Fixed (32.12 Degrees of tilt)
0.512138	Latitude tracking
0.785398 ($= \frac{\pi}{4}$)	Full Tracking

http://www.nku.edu/~longa/solarweb.html

The output of the program includes the information above, plus a “spinning plot”, featuring a sample roof with the specified user-supplied criteria:

* latitude,
* roof pitch,
* and ridge angle (from south).

Why is all this important?

- We get “something for nothing”: we're going to affix the panels to the roof, so why not do it optimally?
- Someday....
  - Roofing materials will be made of solar collectors (some are already, actually!). Pitching the roof appropriately will be a good strategy.
  - More solar collectors are coming on-line all the time, meaning more panels need positioning.
- Why not design buildings so that their pitch is optimal, and coat their roofs with collectors?

Conclusions

- We provide estimates for the improvement in solar energy collection based on “proper pitching” of solar panels, as well as more sophisticated panel placement or tracking.
- Our on-line program assists those seeking to orient their panels, and willing to “pitch them off the roof”, up onto an angle.
- It's not too difficult to add customization, to handle special issues (e.g. mountains, trees shading panels, etc.).
- A little math goes a long way. Look around! Look up! Turns out everything's a nail....