Mat385: a view from the end of the tunnel

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So... you've made it through discrete math! Logic, proofs, induction, recursion, sets, graphs, trees, worst-case scenario analysis, traversals, Boolean algebra, logic networks, finite state machines: admittedly it's only been a taste in most cases, but I hope that you'll remember some of the most important overriding concepts and themes:

- **Minimization/optimization**: mathematics is often about elegance, about making something as simple as possible (but not simpler!). I hope that you will continue to search for the **best** solution for your problems.

  But remember that the first thing is to get the job done. If you do it "right" first, then find a better way, that's great.

  Related:

  a. **There's always more than one way to do things**: the thing that I love most about mathematics is that there is almost always more than one way to do things. Now, being lazy mathematicians, we often seek the easy way (although we may not find it).
  b. But easiest in one way may be harder in another (see recursion, below).

- **Symmetry**: symmetry is elegant, and can be used to save you work! Turn the problem over in your mind, to see if symmetry can help you to solve it. Symmetry may even point the way to a solution.

  I think specifically about duality in Boolean algebra, or in all simple graphs of a certain number of vertices; but also about storage in matrices, in one of our proofs that a Hamiltonian circuit didn't exist in a particular graph, etc.

- **Generalization**: Boolean algebras allow us to talk simultaneously about logic and about set theory. Statements we make in set theory may be proven by a calculation in Boolean algebra; even better, networks that are poorly designed can be optimized by Boolean algebra calculations.

  Related ideas:

  a. **Look for Patterns**: Mathematicians look for patterns, and then try to generalize. Remember Yanghui's (or Pascal's) triangle: the numerical pattern could even inform us about another culture's system of writing numbers. Versions of this triangle exist in many cultures!

b. **Think specifically**: While generalization is wonderful, it may be better to think specifically to see if a general conjecture applies to any particular case. If not, you have a counter-example (if so, then you may have a good concrete context for how to think of a general thing).

   An example of that is to think of "simple graphs" as "Facebook graphs".

- **Recursion**: you already know how important this is, but keep it in the forefront. Often the best way to solve a problem is by solving it over and over and over again, on smaller parts! It's also a remarkably simple way to define things, when possible (e.g. trees, or wffs, or regular expressions).

  Remember the dangers of recursion, however: it may be easy to code up a recursive algorithm, but, as we saw in the case of calculating the n^{th} Fibonacci number, an algorithm that was easy to code up was a bad algorithm computationally, capable of driving a computer to its knees (if it had knees...).

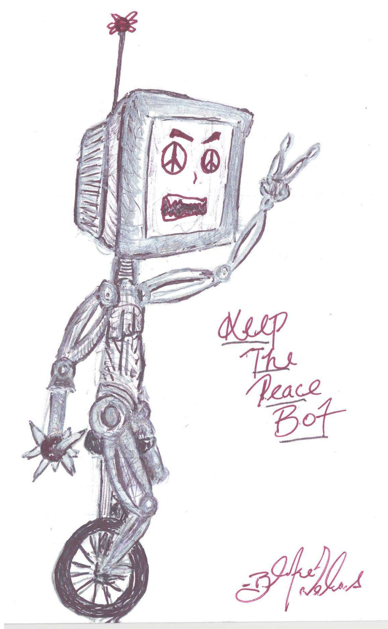

Figure 1: Computers DO have knees (but just two fingers); and this computer appears to be sentient.... Wait – is that ChatGPT? [Thanks to Blake Nelms]

Most of all, remember that mathematics is loaded with tools, the vast majority of which we **didn't** study! Don't stop now: there are plenty of other areas that may prove invaluable to you. If I may recommend two:

- Linear Algebra
- Probability and Statistics

Both of these areas will prove useful to you as you move forward. Consider taking a little of both, if life presents the opportunity. Good luck, in any case!