

Review $\rightarrow (M(J) \rightarrow F(J))$

- | | |
|---|-------------|
| 1. $(\forall x)(M(x) \rightarrow I(x) \vee G(x))$ | hyp |
| 2. $(\forall x)(G(x) \wedge L(x) \rightarrow F(x))$ | " |
| 3. $I(J)$ | " |
| 4. $L(J)$ | " |
| 5. $M(J) \rightarrow (I(J) \vee G(J))$ | 1, $\wedge$ |
| 6. $G(J) \wedge L(J) \rightarrow F(J)$ | 2, $\wedge$ |
| 7. $M(J)$ | ded. method |
| 8. $I(J) \vee G(J)$ | 5, 7, mp |
| 9. $G(J)$ | 3, 8, ds |
| 10. $G(J) \wedge L(J)$ | 4, 9, conj |
| 11. $F(J)$ | 6, 10, mp |

#3 $1 \cdot 2^1 + 2 \cdot 2^2 + \dots + n \cdot 2^n = (n-1) 2^{n+1} + 2$

$\underbrace{\hspace{15em}}_{P(n):}$

Base case: $n=1$

$$\text{LHS: } 1 \cdot 2^1 = 2 \quad \text{RHS: } (1-1)2^{1+1} + 2 = 2 \checkmark$$

Inductive step: $P(k) \rightarrow P(k+1)$

$$P(k): 1 \cdot 2^1 + \dots + k \cdot 2^k = (k-1)2^{k+1} + 2$$

$$P(k+1): \quad \quad \quad + (k+1)2^{k+1} = ((k+1)-1)2^{(k+1)+1} + 2 \\ = k \cdot 2^{k+2} + 2$$

Assume $P(k)$ & consider the LHS of $P(k+1)$:

$$1 \cdot 2^1 + \dots + k \cdot 2^k + (k+1)2^{k+1} = (k-1)2^{k+1} + 2 +$$

$$(k+1)2^{k+1}$$

(by $P(k)$)

$$= k2^{k+1} - 2^{k+1} + 2 + (k+1)2^{k+1}$$

$$= 2k2^{k+1} + 2$$

$$= k2^{k+2} + 2 \checkmark$$

1b, $P \wedge P' \rightarrow Q$

1. P

h.u.n

1, r	hyp
2, P'	hyp
3, Pr Q	1, add
4, Q	2, 3, ds