# MAT229 Test 2 (Spring 2025)

### Name:

**Directions**: Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!** 

**Problem 1 (26 pts):** Consider the definite integral  $I = \int_0^2 \frac{1}{\sqrt[3]{x^2 + 1}} dx$ 

The integrand doesn't have a "nice" antiderivative, so the integral can only be approximated numerically (assume I = 1.5980).

a. (8 pts) Approximate this integral using left endpoint, right endpoint, midpoint and trapezoidal rules, with n = 2. You may use this table of values for the integrand, f(x):

x	0	0.5	1.0	1.5	2.0
f(x)	1.	0.9283	0.7937	0.6751	0.5848

method	estimate	absolute error
LRR		
RRR		
trap		
mid		



b. (4 pts) Derive a Simpson's estimate from part a, and its absolute error. What is its "n" (how many rectangles used)?

c. (2 pts) Do the resulting errors make sense, given the graph of the integrand?

d. **Problem 1**, cont.(12 pts) Using an appropriate error bound, what is the biggest error possible in your midpoint, trapezoidal, and Simpson's estimates? Do your estimates and errors agree with your expectations? You may use the following graphs in your analysis:



## Problem 2 (30 pts):

a. Determine whether the following integrals are convergent or divergent.

i. (10 pts) 
$$J = \int_0^1 \frac{1}{\sqrt{x+x}} dx$$

ii. (10 pts) 
$$K = \int_{1}^{\infty} \frac{1}{x^2 + 1} dx$$

b. (10 pts) Consider the infinite strip given by  $0 \le y \le \frac{x}{(x^2+1)^{3/2}}$  for  $0 \le x < \infty$  (part of which is shown below). Determine if it has finite area (or not).



## Problem 3 (20 pts): Series

- a. Consider the series S having  $n^{th}$  term  $a_n = \frac{1}{n}$ , starting with n = 1.
  - i. (4 pts) Compute the first four partial sums  $S_1, \ldots, S_4$ .

ii. (6 pts) Is the series convergent or divergent, and how do you know?

b. State whether the following series converges or diverges, and explain why. If it converges, give its value.

$$S = 1 - \sqrt{\frac{e}{3}} + \sqrt{\frac{e^2}{9}} - \sqrt{\frac{e^3}{27}} + \dots$$

### Problem 4 (24 pts): Sequences

a. (8 pts) Determine the limit of the sequence  $a_n = \frac{\ln(n^2)}{\ln(2n)}$ , or show that the sequence diverges.

b. (8 pts) Determine whether the sequence  $a_n = \sin\left(\pi + \frac{\pi}{n}\right)$  for  $n \ge 1$  is bounded and whether it is eventually monotone, increasing, or decreasing. If it converges, find its limit.

c. (8 pts) Find an explicit formula for  $a_n$  where  $a_1 = 1$  and  $a_n = a_{n-1} + n$  for  $n \ge 2$ .