

Name:

1. (4 pts) You may recall that $\sinh(x) = \frac{e^x - e^{-x}}{2}$. Use the series $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ to write the Taylor series for $\sinh(x)$ centered at $x = 0$.

(2 pts) Use the ratio test to determine the radius of convergence for the Maclaurin series for $\cosh(x)$:

$$\cosh(x) = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$$

2. (4 pts) You approximate the function

$$e^{-x} = \sum_{n=0}^{\infty} (-1)^n \frac{x^n}{n!}$$

with a fourth degree Taylor polynomial also centered at $x = 0$. Write out the terms in the polynomial, and bound the error of this approximation at $x = 1$ in either of two ways:

- a. the AST error estimate
- b. Taylor's error estimate:

$$R_n(x) \leq \frac{M}{(n+1)!} |x - 0|^{n+1}$$

For one point of extra credit, do it in both ways!