Final Exam

MAT 229, Spring 2021

1. Find an equation of the tangent line to the curve $y = \tan^{-1}(x)$ when $x = 1$.

2. Consider the function $f(x) = x^2 \ln(x)$.

2.1. Write an integral to represent the area A between the graph of $f(x)$ and the x-axis, from $x = 1$ to $x = 2$.

2.2. Use (and demonstrate) integration by parts to compute A.

3. Find the area between the x-axis and the curve $y = \cos^2(x)\sin^3(x)$ for $0 \le x \le \pi/2$. Show all the steps in computing any integrals.

4. Consider the power series $\sum_{n=0}^{\infty} (-1)^{n+1} \frac{(x-5)^n}{3^n}$.

4.1. What is its interval of convergence?

4.2. Rewrite this as a geometric series, and hence write the function as a simple function of x (without the summation).

5. (Taylor polynomials) Function $g(x)$ has the following features

- $g(4) = 1$
- $g'(4) = 2$
- $g''(4) = 3$
- $g^{(3)}(4) = 3$
- $\left|g^{(4)}(x)\right| \le 6$ for $3 \le x \le 5$

5.1. What is the third degree Taylor polynomial for $g(x)$ centered at 4?

5.2. Using the remainder for the third degree Taylor polynomial, what is the maximum error in approximating $g(x)$ with $T_3(x)$ for $3 \le x \le 5$?

6. Let $f(x) = x^2(1 - x)$. Using exactly two equal subintervals and the following methods, compute an approximation to the integral $\int_0^1 f(x)\,dx$, represented in this graph, with values given in the following table. You may use any shortcuts you know.

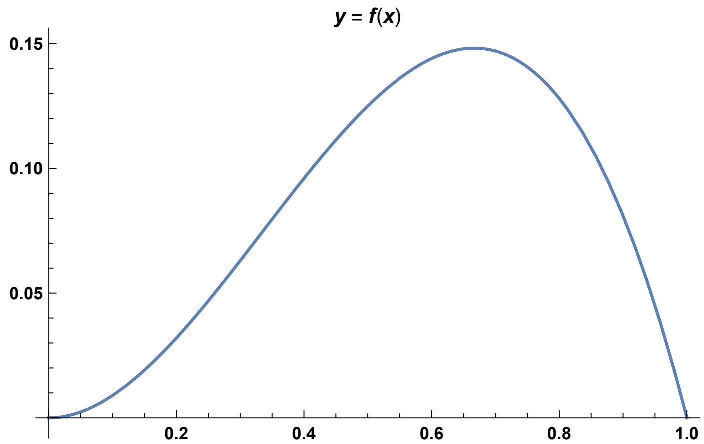

Table of values of $f(x)$.

x	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
$f(x)$	0	$\frac{3}{64}$	$\frac{1}{8}$	$\frac{9}{64}$	0

6.1. First compute the exact value of the integral.

6.2. Now compute the numerical approximations, and fill in a table as follows.

Method	Integral value
Left endpoint	□
Right endpoint	□
Midpoint	□
Trapezoid	□
Simpson	□
Exact	□

7. Determine if the given series converges absolutely, converges conditionally, or diverges. Give reasons for your answer.

7.1. $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k}{\sqrt{3k^2+5}}$

7.2. $\sum_{n=1}^{\infty} \frac{\cos(n)}{1+n^3}$

8. the series $S = \sum_{n=0}^{\infty} \frac{1}{n^2+1}$ is convergent.

8.1. Choose an appropriate improper integral to demonstrate that S is convergent, by using the integral test.

8.2. Write that improper integral as a limit, and evaluate it.

9. Find two sets of polar coordinates for $(x, y) = \left(-\sqrt{3}, 1\right)$, one with $r > 0$ and one with $r < 0$.

10. Consider the parametric equations, given by $x(t) = 3\sin(t)$ and $y(t) = 4\cos(t)$.

10.1. Eliminate the parameter t to write a Cartesian equation for the curve. (It is an equation, so your response must have an equals sign in it.) **Your answer should not include any inverse functions (e.g. arcsin)**.

10.2. Write (but do not evaluate) an integral for the length of the curve over one period.

10.3. Suppose the equations had been $x(t) = 3\sin(t)$ and $y(t) = 3\cos(t)$, instead. The integral becomes easy. Why? What curve does the parametric motion represent?

11. (Vectors) Let $\vec{u} = \langle 1, 0, 2\rangle$ and $\vec{v} = \langle 2, -2, 1\rangle$.

11.1. Determine $2\vec{u} - 3\vec{v}$ in component form.

11.2. Find a unit vector that points in the same direction as $\vec{v}$.

11.3. What is the angle between $\vec{u}$ and $\vec{v}$?

11.4. Find a vector that is perpendicular to both $\vec{u}$ and $\vec{v}$.

12. A plane in three-space may be given by a vector normal to the plane and a single point P_0 lying in the plane: $\langle 1, 2, -1\rangle \cdot (\langle x, y, z\rangle - \langle 3, 7, 8\rangle) = 0$.

12.1. Write this as an equation $z = f(x, y)$.

12.2. Demonstrate that the point (3,6,6) lies in the plane.

12.3. Compute the components of the vector from the point P_0 to (3,6,6). Demonstrate that this vector is perpendicular to the vector normal.