a. $\lim\limits_{x \to 0} \dfrac{\overbrace{\ln(1+x)}^{f(x)}}{\underbrace{x}_{g(x)}} = \lim\limits_{x \to 0} \dfrac{\frac{1}{1+x} \cdot (1+x)'}{1}$

$$= \lim_{x \to 0} \frac{\frac{1}{1+x}}{1}$$

$$= \lim_{x \to 0} \frac{1}{1+x} = 1$$

$$\left( = \frac{f'(0)}{g'(0)} = \frac{1}{1} = 1 \right)$$

b. $\lim\limits_{x \to \pi} \dfrac{\cos(x)}{x} = \dfrac{\lim\limits_{x \to \pi} \cos(x)}{\lim\limits_{x \to \pi} x} = \dfrac{-1}{\pi}$

c. $\lim_{x \to 1} \frac{2\ln(x)}{1-e^{x-1}} = \lim_{x \to 1} \frac{f(x)}{g(x)}$

[Is it indeterminate?]

Yes! Use L'Hôpital:

$f'(x) = 2 \cdot \frac{1}{x} = \frac{2}{x}$

$g'(x) = \left(1 - e^{x-1}\right)'$

$= -\left(e^{x-1}\right)'$

$= -e^{x-1} \cdot (x-1)'$

$= -e^{x-1} \cdot 1$

$= -e^{x-1}$

$\lim_{x \to 1} \frac{2\ln(x)}{1-e^{x-1}} = \frac{f'(1)}{g'(1)} = \frac{2}{-1} = -2$

d. $\lim_{x \to 0} \frac{\sin(x) - x}{\cos(2x) - 1} = \lim_{x \to 0} \frac{f(x)}{g(x)}$

[Is it indeterminate?]

Yes - use L'Hôpital!

$f(x) = \sin(x) - x$

$f'(x) = \cos(x) - 1$

$f'(0) = 1 - 1 = 0$

$g(x) = \cos(2x) - 1$

$g'(x) = (\cos(2x) - 1)'$

$= (\cos(2x))'$

$= -\sin(2x) \cdot 2$

$g'(0) = 0$

$\lim_{x \to 0} \frac{\sin(x) - x}{\cos(2x)} = \lim_{x \to 0} \frac{\cos(x) - 1}{-2\sin(2x)}$

$= \frac{-\sin(0)}{-4\cos(2 \cdot 0)} = \frac{0}{1} = \boxed{0}$