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MAT 375

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1. Compare the models: \* years starting at 70

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|  | **Linear Model** | | **Exponential Model** | | **Power Model** | |
| **Model Fit** | AdExp = -401900 + 5809.16\*years | | AdExp = 11.4379\*eyears\*0.105489 | | AdExp = 6.83224\*10-12 \* years8.35392 | |
| **R2** | 0.95307 | | 0.986956 | | 0.992494 | |
| **Parameter CIs (95%)** | a | (-452781, -351019) | a | (8.16543, 16.02211) | a | (1.51516\*10-12, 3.08002\*10-11) |
| b | (5170.82, 6447.49) | b | (0.101261, 0.109717) | b | (8.00963, 8.69822) |
| **Fitted Plot** |  | |  | |  | |
| **Residuals** |  | |  | |  | |

The three models all perform relatively well, but some deal with issues posed by residuals better. The linear model achieves an R-squared of 95.3%, which is very high, but both the exponential and power models outperform it with R-squareds of 98.7% and 99.2%, respectively. This value is not largely different for the latter two models, so we must examine the fitted plot and graphs of residuals to determine which model is better. The power model appears to do a better job of meeting the assumption of normality in the residuals, as can be seen in the histograms. From the residual vs. time plots, both the power and exponential model still suggest that the residuals are not entirely independent. There appears to be some s-curvature in the data that neither of these models addresses.

1. What problem does the power model pose, relative to starting value of time t? Does it change if we represent years as 1970 rather than 70?

The power problem poses a problem relative to the starting value of time. Time itself is the base of the model, with a value “b” called the “power” applied to it. This parameter determines the function’s growth rate and general shape. The y-intercept of the power model is always going to be 0, because time is 0, so the model itself will extend to zero whether we start at 70 or 1970. Because the function will need to appear “dormant” for a much longer time if we start at 1970, this will force the growth rate to be much larger so that the appropriate growth can be achieved in a proportionally smaller amount of time than if we had started at 0. To stop this growth from getting out of control too early on, an extremely small “scaling factor”, which is the “a” parameter, becomes necessary. The power model if we start at 70 is:

AdExp = 6.83224\*10-12 \* years8.35392

The model if we start at 1970 is:

AdExp = 2.1047\*10-684 \* years208.817

 

Figure 1: Model starting at 70 extended to 0 (1900) Figure 2: Model starting at 1970 extended to 0

The graphs of the two will appear identical if the x-axis is specified to the respective year ranges, however we must remember that the graph extends all the way to 0. This means the latter function will be hugging 0 for a very long time (see graphs above). If we look at values inside our range, the models show perform identically. However, the second we extrapolate, we run into problems. The scaling factors out front are calculated using the given data, and would quickly be overpowered by the power/growth rate. For example, if we wanted to look 5 years into the future, the model starting at 1970 would predict a value much higher for 1994 than the model starting at 70 would predict for 94. This is because we’ve inflated the power by an extraordinary amount in order to achieve the “fit” or “shape” we desired for our data. Therefore, the model starting at 70 would make more sense to use as it doesn’t distort the model as much (and we don’t really care about advertising expenditures at the time of Christ anyway).

1. How do we interpret the power model? (Remember my remarks above about interpreting the parameters of your model!)

The power model has two parameters: a, the scaling factor, and b, the power. The scaling factor in both cases is extraordinarily close to 0, and the effect this has on the model is that the fit is “pulled down”. Otherwise, advertising expenditures would shoot up rapidly beginning at early years in our model, which is not the case. The particularly interesting parameter, however, is the exponent of 8.35392. This value corresponds to the growth rate, as it is larger than 1, of our model. This indicates that as time goes on, advertising expenditures increase faster and faster. This makes sense in context because if we assume that a company grows with time, then we can also assume that the company will have more and more income to use for different purposes, such as advertising, with each passing year.

1. Does the exponential model suffer the same problem? What is the impact of a shift in time scale?

The exponential model does not suffer from the same problem. A shift in time scale (i.e. from 0 to 1900) will impact the leading coefficient, and thus the intercept to reflect the starting point. However, the base of the function and the growth parameter, e and “b” (in this case 0.105489), do not change. This means that both models would share the same fit and growth rate, which would lead to approximately equal predictions. When the time scale shifted for the power model, the growth rate skyrocketed, and this leads to very different predictions between those two models.