

Directions: Show your work! Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put it in the box provided). **Good luck!**

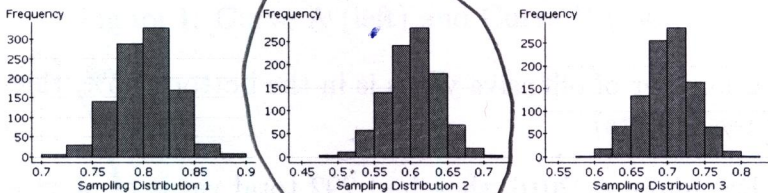
Problem 1: (20 pts)

a. Give the symbol that commonly corresponds to each of the following: (1 pt each)

Quantity:	Mean	Standard Deviation	Population Proportion	Sample Proportion	Sample Size
Symbol:	μ	σ	P	\hat{p}	n

$p = .60$
b. A recent report stated that 60% of all adults in a community support a ban on texting while driving. A cell phone company investigates this percentage by taking a random sample of $n = 200$ adults from voter registration records.

1. Which of the graphs below would be the correct sampling distribution for the proportion of adults holding this opinion when samples of size 200 are taken? Circle the label underneath the correct one, and explain your choice. (5 pts)



The graph shows that at $p = .6$, the frequency is the highest.

2. Identify the following from the study above (1 pt each):

Population	adults
Sampling frame	voter registration records
Sample statistic	$p = .60$
Variable of Interest	opinion of texting while driving (support/not support)
Type of Variable of Interest	Categorical (Y/N)

3. Describe the shape of the barplot you circled above, and explain why it has the shape it does. (5 pts)

Normal distribution. To determine if the shape is normal...

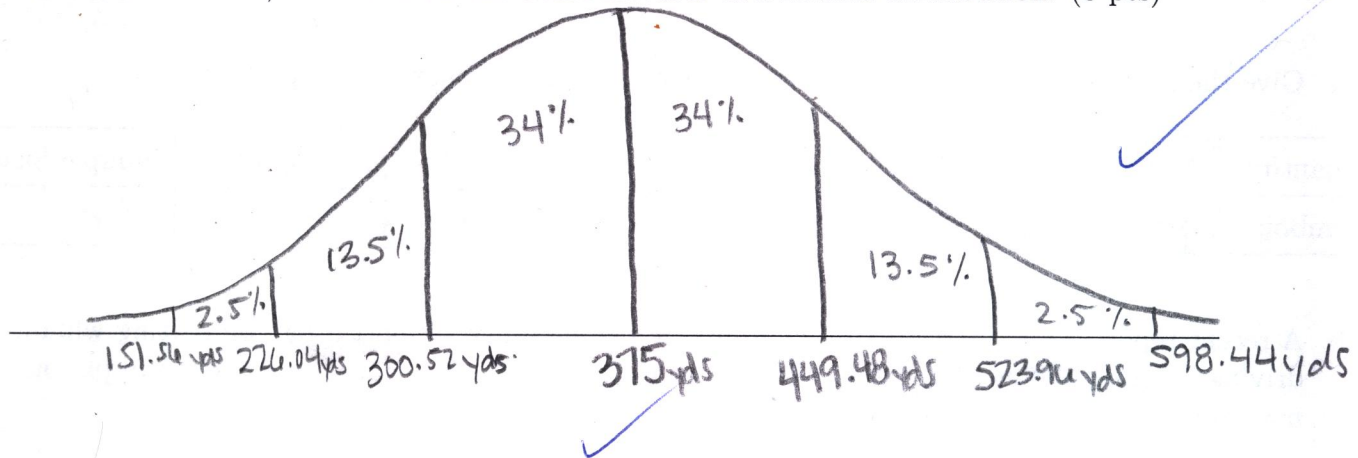
$$n(1-p) = 200(1-.60) = 80 > 10 \checkmark$$

$$np = 200(.60) = 120 > 10 \checkmark$$

If the value is greater than 10 ($x > 10$) then normality can be assumed

Problem 2: (20 pts) In the 2015 NFL season, the total number of offensive yards for the winning team was approximately normally distributed with a mean of 375 yards and a standard deviation of 74.48 yards. In the off-season, a coach would like to use this information to make predictions for the upcoming season. Use the Empirical Rule to answer parts b-d; you'll need your z-table for parts e and f.

- a. Draw the corresponding normal distribution. Make sure that you capture the mean and the standard deviation, as well as all the usual features of a normal distribution. (6 pts)

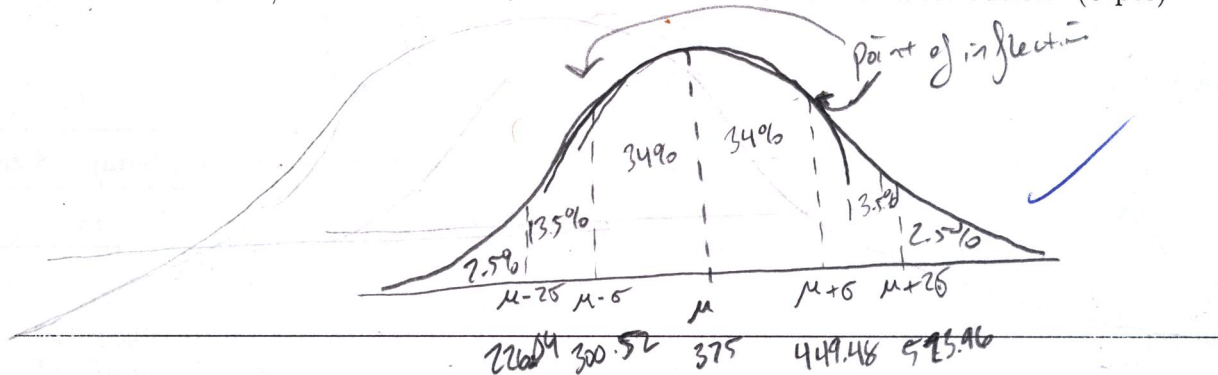


- b. We expect the winning team in 95% of NFL games to have a total number of offensive yards between 226.04 yds and 523.96 yds. (4 pts)
- c. The percentage of winning teams having 449.48 total offensive yards or less is 84%%. (2 pts)

$$\mu = 375 \quad \sigma = 74.48$$

Problem 2: (20 pts) In the 2015 NFL season, the total number of offensive yards for the winning team was approximately normally distributed with a mean of 375 yards and a standard deviation of 74.48 yards. In the off-season, a coach would like to use this information to make predictions for the upcoming season. Use the Empirical Rule to answer parts b-d; you'll need your z-table for parts e and f.

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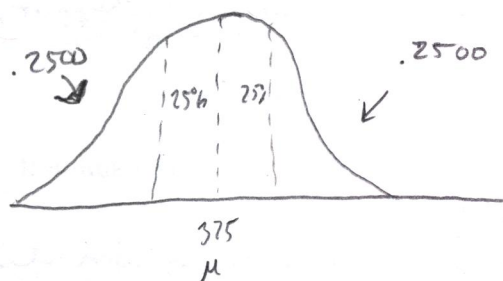


- b. We expect the winning team in 95% of NFL games to have a total number of offensive yards between 226.04 and 523.96. (4 pts)
- c. The percentage of winning teams having 449.48 total offensive yards or less is 84%. (2 pts)
- d. If a winning team's total number of offensive yards is in the bottom 2.5%, then the team had 226.04 yards or less. (2 pts)
- e. When Cincinnati beat Baltimore in 2016, they had 292 total yards of offense. This total is 1.11 standard deviations to the left of the mean. (3 pts)

$$Z = \frac{292 - 375}{74.48} = -1.11$$

probability of .1335 or 13.35%

- f. What offensive yardages correspond to the middle half (50%) of all winning totals? (3 pts)



$$-0.67 = \frac{x - 375}{74.48}$$

$$0.67 = \frac{x - 375}{74.48}$$

$$-49.9016 = x - 375$$

$$x = 325.0984$$

$$325.1 \text{ to } 424.9 \text{ yds.}$$

$$0.2514 = \dots$$

$$Z = -0.67$$

$$49.9016 = x - 375$$

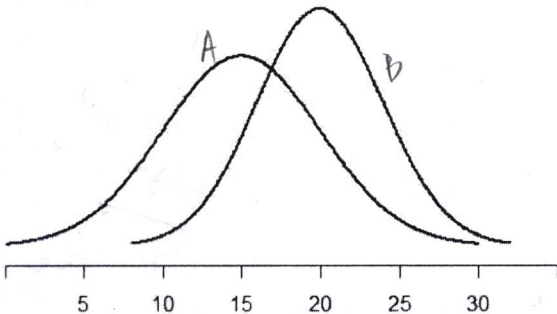
$$x = 424.9016$$

Problem 3: (16 pts) For each statement below either circle the most appropriate answer, or fill in the blanks. (2 pts each)

- a. ~~True~~ **False**: The margin of error for a confidence interval increases as the sample size increases.
- b. **True** ~~False~~: A 92% confidence interval is narrower than a 97% confidence interval.
- c. **True** ~~False~~: When calculating the sample size needed to achieve a given margin of error, a sample proportion of 0.5 will maximize the sample size required.
- d. **True** ~~False~~: When a hypothesis test is conducted based on sample data, the probability of a Type II error is the significance level α .
- e. **True** ~~False~~: If a Type I error is of greater concern than a Type II error, we set α to be a smaller number (such as 0.01).
- f. **True** ~~False~~: A parameter is used to estimate a statistic.
- g. **True** ~~False~~: A convenience sample of 200 is better than a random sample of 100.
- h. A proportion is a number between 0 and 1.

Problem 4: (8 pts) Two normal curves are given. Use these pictures to determine the following:

Figure 1: Curve A (left) and Curve B (right)



- a. Select the nearest integer values:

The mean of curve A is 15. The mean of curve B is 20. (3 pts)

- b. Select from the following values: 4, 5, 10, 12:

The standard deviation of curve A is 5. The standard deviation of curve B is 4. (3 pts)

- c. If we were to calculate a z-score for $y=16$ for each of the two curves, Curve A would result in a positive z-score and Curve B would result in a negative z-score. (2 pts)

Problem 5: (12 pts) In a large company, 60% of its employees are men. It has recently come under fire for the number of males in executive-level positions, and the claim has been made that the proportion of men in executive-level positions differs from the proportion of men who work for the company. Answer the following, **with some justification**. (2 pts each)

- a. Write the hypotheses which would be used to test the claim made.

$$H_0: p = 0.60 \quad H_a: p \neq 0.60$$

- b. When the sample is taken, the test statistic is found to be $z = 1.31$. What probability would be used to test the hypotheses written in part a?

Circle one: 0.0951 0.9049 0.1902 1.8098

because it is a two tail probability $p \neq .60$ so we have to use both side

- c. Using a significance level of 0.10, what decision would be made in the test?

Circle one: reject H_0 fail to reject H_0

because probability = 0.1902 > 0.10

- d. Based on the decision made in part c, what type of error is possible?

Circle one: Type I error Type II error

α is big, and we fail to reject H_0

↑
false

- e. What would be the consequence to the company of the error you identified in part d.?

This company may stay with the same number of men in the executive-level thinking it is balance. It can come under fire again

- f. Describe how both descriptive and inferential statistics are being used in this problem.

We looked at the information and summerize it

We took a small group and try to expend it to the whole

N. e

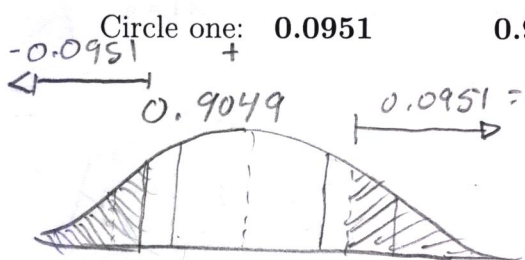
Problem 5: (12 pts) In a large company, 60% of its employees are men. It has recently come under fire for the number of males in executive-level positions, and the claim has been made that the proportion of men in executive-level positions differs from the proportion of men who work for the company. Answer the following, **with some justification**. (2 pts each)

a. Write the hypotheses which would be used to test the claim made.

$$H_0: p = 0.60$$

$$H_A: p \neq 0.60$$

b. When the sample is taken, the test statistic is found to be $z = 1.31$. What probability would be used to test the hypotheses written in part a?



we are saying it is not equal to $p = 0.60$, so it could be more than or less than so we need both areas colored in on the left

c. Using a significance level of 0.10, what decision would be made in the test?

Circle one: reject H_0

fail to reject H_0 .

if $p < 0.10$ reject null or H_0 .

$$0.1902 > 0.10$$

accept H_0

d. Based on the decision made in part c, what type of error is possible?

Circle one: Type I error

Type II error

reject a true null

fail to reject a false null

e. What would be the consequence to the company of the error you identified in part d?

if it is true that $p \neq 0.60$ and the amount of employees that are men are not around 60% and the company believes that it is 60% then they might hire more men when they got too many or fire men when they do not have enough!

f. Describe how both descriptive and inferential statistics are being used in this problem.

Descriptive - summarizes and generalizes data while inferential draws a generalization about a population from a sample statistic. We are using the data inferentially to decide whether the company has too many men while we are using the data descriptively to show how (roughly) many men work for the company.

Problem 6: (20 pts) A hot topic in the news is the possible replacement of the Brent Spence Bridge, connecting northern Kentucky and Cincinnati. If the bridge is replaced, it will likely become a toll bridge. A 2013 sample of 1,675 randomly selected AAA members in northern Kentucky was asked their opinion on instituting a toll on the Brent Spence Bridge. Of those surveyed, 988 were opposed to tolls.

- a. Are you justified in using normal statistics to explore this problem? (2 pts)

Yes because the sample is random and the sample size is ~~over 10~~

$$\begin{aligned} np &= 1675(.5899) = 988.08 \\ n(1-p) &= 1675(.4111) = 688.59 \end{aligned} \quad \left. \begin{array}{l} \text{Both are over 10} \end{array} \right\}$$

- b. Estimate the percentage of northern Kentucky residents who are opposed to tolls on the bridge with 95% confidence. (10 pts)

$\rightarrow z$

$$n = 1675 \quad z = 1.96$$

$$\hat{p} = \frac{988}{1675} = .5899$$

$$\text{Solve: } \hat{p} \pm z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$.5899 \pm 1.96 \sqrt{\frac{.5899(.4111)}{1675}}$$

$$.5899 \pm .0236$$

$$.5899 - .0236 = .5663$$

$$.5899 + .0236 = .6135$$

$$\left. \begin{array}{l} .5663 \\ .6135 \end{array} \right\} [.5663, .6135]$$

Conclusion: With 95% confidence, we estimate that the number of Northern Kentucky residents who are opposed to tolls on the bridge is between 56.63% and 61.35%.

- c. Does the interval found in part b. provide evidence that less than 60% of residents are opposed to tolls? Circle one: yes (no) Explain. (3 pts)

Because the interval only shows that the lowest percentage is less than 60% not the highest. The highest percentage is over 60%.

Problem 6: (20 pts) A hot topic in the news is the possible replacement of the Brent Spence Bridge, connecting northern Kentucky and Cincinnati. If the bridge is replaced, it will likely become a toll bridge. A 2013 sample of 1,675 randomly selected AAA members in northern Kentucky was asked their opinion on instituting a toll on the Brent Spence Bridge. Of those surveyed, 988 were opposed to tolls.

$$n = 1675 \quad \hat{p} = 988/1675 = 0.5899$$

a. Are you justified in using normal statistics to explore this problem? (2 pts)

random sample ✓

$$n \cdot p = 988.08 > 10 \quad \checkmark$$

$$n \cdot (1-p) = 686.92 > 10 \quad \checkmark$$

Yes because it was a random sample and it is a normal distribution. ✓

b. Estimate the percentage of northern Kentucky residents who are opposed to tolls on the bridge with 95% confidence. (10 pts)

$$z = 1.96$$

$$\hat{p} = 0.5899$$

$$n = 1675$$

$$\hat{p} \pm z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.5899 \pm 1.96 \sqrt{\frac{0.5899(1-0.5899)}{1675}}$$

$$0.5899 \pm 0.0236$$

$$[0.5663, 0.6135] \quad \checkmark$$

With 95% confidence, between 56.63% and 61.35% of northern Kentucky residents are opposed to tolls on the bridge.

c. Does the interval found in part b. provide evidence that less than 60% of residents are opposed to tolls? Circle one: yes **no.** Explain. (3 pts)

No, because the upper interval is above 60%, so we can't conclude that less than 60% of residents are opposed to tolls.

- d. (Problem 6, cont.) State legislators would like to conduct a new survey to update the estimate for 2015. They would like to increase the level of confidence to 98% with a margin of error of + 2.25%. If they were to use the point estimate in 2013 as a starting point, how many people should be included in the 2015 sample? (5 pts)

$$n = \left(\frac{z}{MOE} \right)^2 \hat{p}(1-\hat{p})$$

$$n = \left(\frac{2.326}{.0225} \right)^2 .59(1-.59)$$

$$n = 2585.2 \text{ people}$$

↓ round up

$$n = 2586 \text{ people}$$

- e. Suppose the legislators merely suspect that opposition to tolls on the bridge has gone up (use the point estimate from 2013 as p). They sample 1000 people and find that 620 oppose tolls. Test their hypothesis at the $\alpha = .05$ level. (4 pts)

$$\hat{p} = 620/1000 \quad \hat{p} = .62 \quad n = 1000$$

$$H_0: p = .59$$

$$H_A: p > .59$$

$$z = \frac{.62 - .59}{\sqrt{\frac{.59(1-.59)}{1000}}}$$

$$z = 1.93 \rightarrow .9732$$

↓

$$1 - .9732 = 0.0268 < \alpha = .05 \quad \checkmark$$

It is below the given α level, meaning the H_0 is rejected in favor of the H_A . This means that opposition to tolls on the bridge has gone up.

- d. (Problem 6, cont.) State legislators would like to conduct a new survey to update the estimate for 2015. They would like to increase the level of confidence to 98% with a margin of error of + 2.25%. If they were to use the point estimate in 2013 as a starting point, how many people should be included in the 2015 sample? (5 pts)

$$Z = 2.326$$

$$\hat{p} = .5899$$

$$MOE = .0225$$

$$n = \left(\frac{Z}{MOE} \right)^2 \hat{p} (1 - \hat{p})$$

$$n = \left(\frac{2.326}{.0225} \right)^2 .5899 (1 - .5899)$$

$$(10686.965) .5899 (1 - .5899)$$

$$2585.369$$

$$n = 2586 \quad \checkmark$$

- e. Suppose the legislators merely suspect that opposition to tolls on the bridge has gone up (use the point estimate from 2013 as p). They sample 1000 people and find that 620 oppose tolls. Test their hypothesis at the $\alpha = .05$ level. (4 pts)

$$\frac{620}{1000} = .62$$

$$\hat{p} = .62$$

$$\text{Step 1: } H_0: p = .62$$

$$H_a: p > .62$$

$$\text{Step 2: } \alpha = .05 \quad \text{We reject } H_0, \text{ if } p < \alpha = .05.$$

$$\text{Step 3: } \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{.62 - .5899}{\sqrt{\frac{.5899(1 - .5899)}{1000}}} = \frac{.0301}{\sqrt{.000202}} = Z = 1.94 \quad \checkmark$$

$$1 - .9738 = .0262$$

Step 4: There is significant evidence to conclude that we reject the H_0 since $p(.0262)$ is $< \alpha = .05$. There is significant evidence to support that opposition to tolls on the bridge has gone up. ✓

Nice