Lagrange Polynomial Approach

To fit (x_1, y_1), (x_2, y_2), (x_3, y_3),
use

$$q(x) = y_1 \frac{(x-x_2)(x-x_3)}{(x_1-x_2)(x_1-x_3)} + y_2 \frac{(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_3)} + y_3 \underbrace{\frac{(x-x_1)(x-x_2)}{(x_3-x_1)(x_3-x_2)}}_{\text{fundamental polynomial}}$$

$$l_3(x) = \begin{cases} 1 & x = x_3 \\ 0 & x = x_1,\ x = x_2 \end{cases}$$

Newton's Polynomial Approach

To fit (x_1, y_1), (x_2, y_2), (x_3, y_3)

$$p_0(x) = y_3$$

$$p_1(x) = p_0(x) + \frac{y_2 - p_0(x_2)}{x_2 - x_3}(x - x_3)$$

$$p_2(x) = p_1(x) + \frac{y_1 - p_1(x_1)}{(x_1 - x_2)(x_1 - x_3)}(x - x_2)(x - x_3)$$

basic idea:

$$p_n(x) = p_{n-1}(x) + \text{correction}$$

(p. 211)

$$= f[x_p] + f[x_1, x_2](x - x_1) + f[x_1, x_2 x_3](x - x_1)(x - x_2)$$

Horner's Polynomial Approach

$$h(x) = y_3 + b(x-x_3) + a(x-x_3)^2$$

$$h(x_1) = y_3 + b(x_1-x_3) + a(x_1-x_3)^2 = y_1$$

$$h(x_2) = y_3 + b(x_2-x_3) + a(x_2-x_3)^2 = y_2$$

Define $d_{ij} \equiv x_i - x_j$

$$b d_{13} + a d_{13}^2 = y_1 - y_3$$

$$b d_{23} + a d_{23}^2 = y_2 - y_3$$

$$b + a d_{13} = \frac{y_1 - y_3}{x_1 - x_3} \equiv f[x_3, x_1]$$

$$b + a d_{23} = \frac{y_2 - y_3}{x_2 - x_3} \equiv f[x_3, x_2]$$

Subtract

<u>first</u> divided differences

$$a(\underbrace{d_{13}-d_{23}}_{d_{12}}) = f[x_3,x_1] - f[x_3,x_2]$$

$$a = \frac{f[x_3,x_1] - f[x_3,x_2]}{x_1 - x_2} \qquad (p\ 215)$$

$$\boxed{a = f[x_2,x_3,x_1]} \qquad \left(\begin{array}{c}\text{second} \\ \text{divided} \\ \text{difference}\end{array}\right)$$

$$\boxed{b = f[x_3,x_2] - a(x_2 - x_3)}$$

We're done -

$$h(x) = y_3 + b(x-x_3) + a(x-x_3)^2$$