MAT129 Test 1 (Spring 2016): Functions, Limits, and Derivatives

Name:

Directions: All problems are equally weighted. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 0. Definitions and theorems

a. Give any of the versions of "the most important definition in calculus" – the limit definition of the derivative.

b. Give a technically correct definition of a function f continuous at the point $x = a$.

c. Draw a diagram and give an explanation of the squeeze theorem. Include assumptions and conclusion.

d. Draw a diagram and give an explanation of the intermediate value theorem. Include assumptions and conclusion.

Problem 1. Consider $f(x) = \cos(\pi x) - \sqrt{x} + x^2 - 3$. **Justify** your answers to the following:

a. How would you describe the function f: is it in a particular class that you recognize? What kinds of functions make it up?

b. What is its domain of definition?

c. Where is it continuous? If it is discontinuous at a point, describe the type of discontinuity.

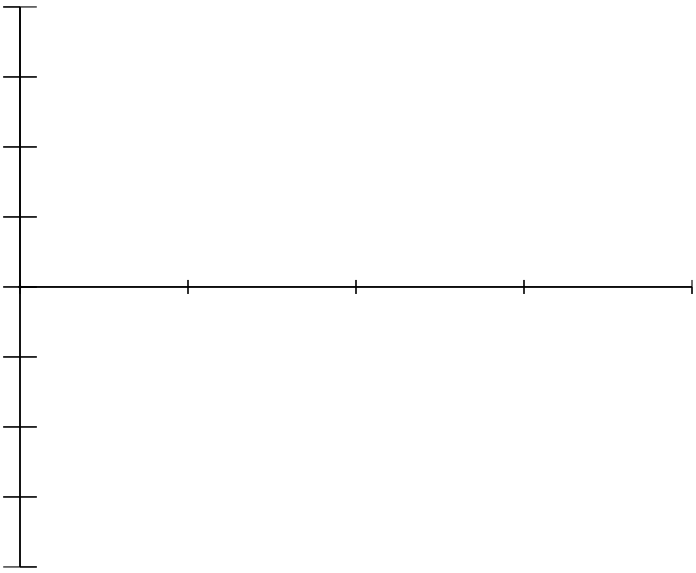
d. Use basic limit laws to find $\lim_{x \to 1} f(x)$ (citing each as you use it).

Problem 2. Consider the function $f(x) = x^2 - 2x - 1$.

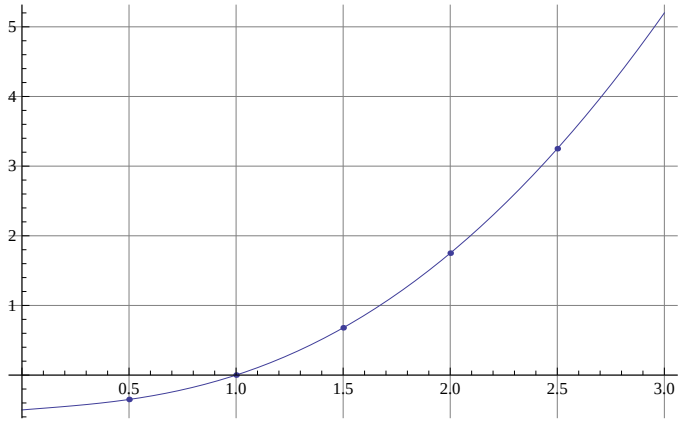
a. (4 pts) Use **the limit definition** to compute the derivative $f'(2)$.

b. (3 pts) Use this derivative to write the equation of the tangent line to the function at $x = 2$.

c. (3 pts) **Carefully** graph both f and the tangent line on this axis (use the interval $[0, 3]$ for your domain):



Problem 3. Consider the function $f(x) = \dfrac{x^3 - 1}{x + 2}$, shown in the graph below:



a. Use both the graph and the formula for f to calculate/estimate the **average** rate of change of f over the interval $[0, 3]$. Show your work on the graph.

b. Estimate the **instantaneous** rate of change of f at $x = 2$. You may use the graph or the formula.

Problem 4. Let $f(x) = \dfrac{x^3 - x + 1}{x - 1}$.

a. (6 pts) Given only that $\lim_{x \to c} a = a$, $\lim_{x \to c} x = c$, and the sum, product, and quotient limit laws, find $\lim_{x \to 2} f(x)$, citing the appropriate limit laws as you go. **You may only use these limit laws.**

b. (2 pts) Explain why it is permissible to compute $\lim_{x \to 2} f(x)$ by substitution.

c. (2 pts) Demonstrate that your answer is correct, using numerical calculations.

x	f(x)

Problem 5. Create the graph of a function f on the interval $(-2, 6)$ that has the following properties:

a. Limits from the left and right as $x \to 0$ are different, but f is continuous from the right at 0;

b. f is continuous for $x > 0$.

c. $x = 3$ and $x = 5$ are the only places that $f(x) = 2$.

d. The derivative $f'(3)$ is positive, while the derivative at $x = 5$ is negative.

e. $\lim_{x \to -1} = \infty$.

f. f is not differentiable at $x = 4$.