

Section Summary: 4.3

1. Definitions

- **Fresnel function:**

$$S(x) = \int_0^x \sin(\pi t^2/2) dt$$

Named after Augustin Fresnel (1788-1827), famous for his work in optics. The Fresnel lens was a miraculous improvement in lighthouse technology: it allowed them to cast a much more powerful beam, and saved many ships and sailors from Davy Jones's locker....

2. Theorems

- **Fundamental Theorem of Calculus, Part I**

If f is continuous on $[a, b]$, then the function g defined by

$$g(x) = \int_a^x f(t)dt \quad a \leq x \leq b$$

is continuous on $[a, b]$ and differentiable on (a, b) , and $g'(x) = f(x)$.

g is undoubtedly one of the oddest functions we have seen so far: it is defined as an integral. Symbolically we might write

$$g : x \longrightarrow \int_a^x f(t)dt$$

It is a map from the real numbers, represented by variable x , into the real numbers (the values of the integral).

- **Fundamental Theorem of Calculus, Part II**

If f is continuous on $[a, b]$, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

where F is any antiderivative of f : that is, a function such that $F' = f$.

The big news is that if you know an antiderivative of integrand f , then computing an integral is easy.

- **Fundamental Theorem of Calculus**

Suppose f is continuous on $[a, b]$.

(a) If $g(x) = \int_a^x f(t) dt$, then $g'(x) = f(x)$.

(b)

$$\int_a^b f(x)dx = F(b) - F(a)$$

where F is any antiderivative of f .

3. Properties/Tricks/Hints/Etc.

- An important example of this relationship between the function and its antiderivative is given from the physics of integrating velocity:

$$\int_a^b v(t)dt = s(b) - s(a)$$

the result is the position function.

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$$\frac{d}{dx} \int_a^x f(t)dt = f(x)$$

4. Summary

This section introduces the fundamental theorem of calculus. It contains two parts: it shows that integrals are solved using antiderivatives, and that derivatives of functions defined using variable limits are solved using derivatives:

Suppose f is continuous on $[a, b]$.

(a) If $g(x) = \int_a^x f(t) dt$, then $g'(x) = f(x)$.

(b)

$$\int_a^b f(x)dx = F(b) - F(a)$$

where F is any antiderivative of f .

The key is “variable limits”: for these functions, the variable is in the **limits** of integration (and **not in the integrand**). One problem, or common reason for misunderstanding, is that there is a “dummy variable of integration” in the problem (no offense!). The variable t in the integral

$$\frac{d}{dx} \int_a^x f(t)dt = f(x)$$

is a **dummy** variable: you notice that t doesn’t appear on the right hand side: only x appears, because t has disappeared during integration.