MAT128 Test 1 (Fall 2015): Functions, Limits, and Derivatives

Name:

Directions: All problems are equally weighted. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it).

Good luck!

Problem 0 (15 pts).

a. (5 pts) Give a technically correct definition of a function.

b. (5 pts) What is the important definition in calculus (according to Long)?

c. (5 pts) Describe accurately the relationship between continuity and differentiability. (Figures are encouraged.)

Problem 1 (20 pts). Consider $f(x) = \dfrac{\sqrt{x}}{(\sin(x))^2 - 1}$. Justify your answers to the following:

a. (8 pts) How would you describe the function f? Is it in a particular class that you recognize? What classes of functions make it up? Use terminology from class.

b. (6 pts) What is its domain of definition?

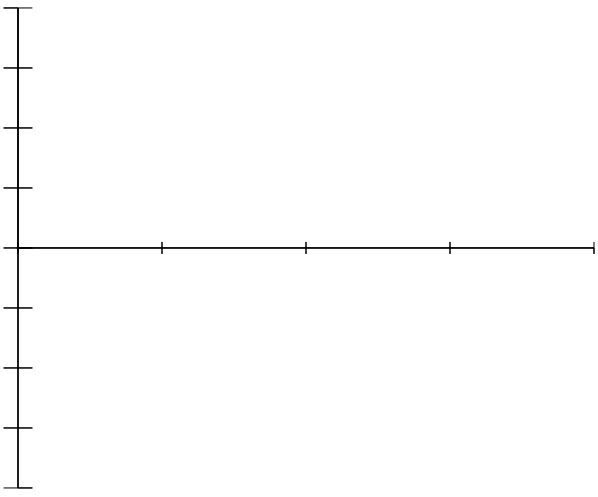
c. (6 pts) Where is it continuous? If it is discontinuous at a point, describe the type of discontinuity.

Problem 2 (25 pts). Consider the function $f(x) = x^2 - 3x + 1$.

a. (10 pts) Use the limit definition to compute the derivative $f'(2)$.

b. (5 pts) Use this derivative to write the equation of the tangent line to the function at $(2, f(2))$.

c. (10 pts) Graph both f and the tangent line well on this axis:



Problem 3 (15 pts). Let $f(x) = \dfrac{x^2 - 1}{x + 2}$.

a. (10 pts) **Given only** that

- $\lim_{x \to c} a = a$,
- $\lim_{x \to c} x = c$, and
- the sum, product, and quotient limit laws,

find $\lim_{x \to 2} f(x)$, **citing the appropriate limit laws as you go**.

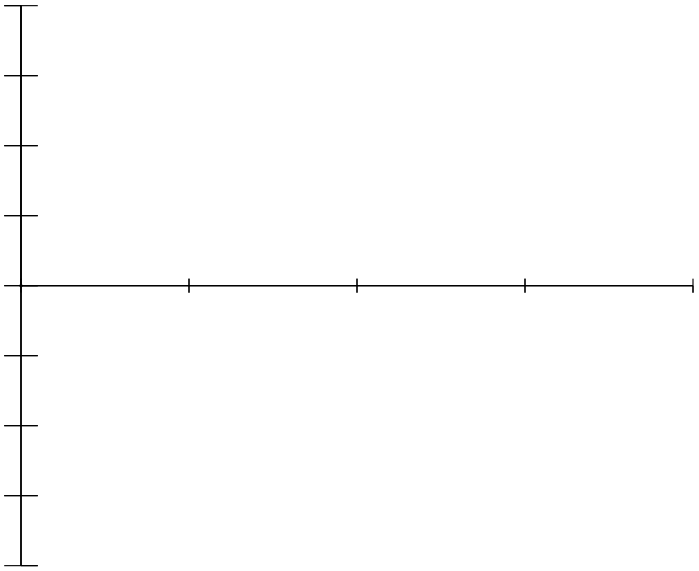
b. (5 pts) Now explain why you can do this limit by substitution, using general principles about classes of functions.

Problem 4 (25 pts). Water drains from a tank, given by the following heights (in meters) over time (in minutes):

time t (minutes)	0	1	2	3	4
height $h(t)$ (meters)	4	2	1	0.5	0.25
average rate of change	NA				

a. (10 pts) Using the data in the table above, choose two points to create a secant line whose slope approximates the derivative of h at $t = 2$.

b. (10 pts) Graph the data from the table, the secant line you used, and then your guess for the tangent line to the function at $t = 2$.



c. (5 pts) What is the average rate of change over each minute? Make a reasonable guess. Add your estimates to the table above (where each answer in the box represents the average rate of change for that minute).