MAT115H Exam 1 (Fall 2015)

Name:

Directions: Attempt all problems, and show your work! Your written work allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning, or a good start). Indicate clearly your answer to each problem (e.g., put a box round it).

Problem 1: (20 pts) Your job in this problem is to compute Moran's I for a graph. Recall that

$$I = \frac{N}{\sum_i \sum_j w_{ij}} \frac{\sum_i \sum_j w_{ij}(X_i - \bar{X})(X_j - \bar{X})}{\sum_i (X_i - \bar{X})^2}$$

Compute Moran's I when the data is $\begin{bmatrix} -1 \\ -1 \\ 1 \\ 1 \end{bmatrix}$, and the adjacency matrix is $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$.

a. $N =$

b. $\sum_i \sum_j w_{ij} =$

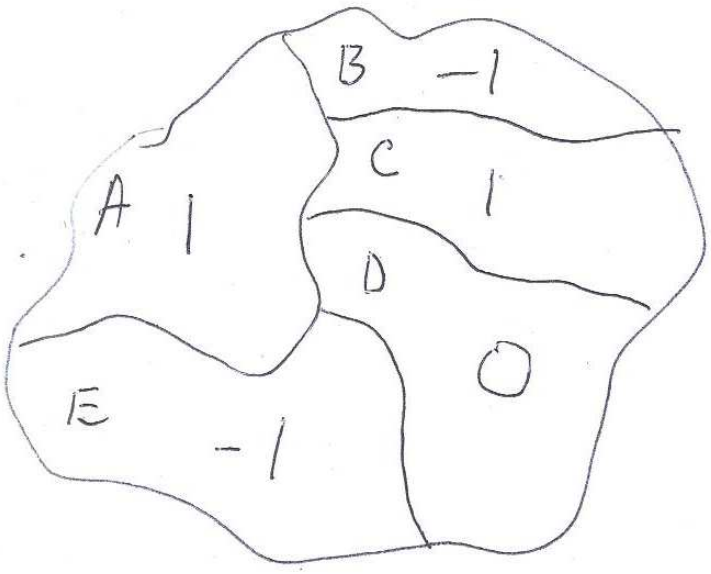
c. $\overline{X} =$

d. $\sum_i \left(X_i - \overline{X}\right)^2 =$

e. $\sum_i \sum_j w_{ij} \left(X_i - \overline{X}\right)\left(X_j - \overline{X}\right) =$

f. $MoransI =$

Problem 2: (10 pts) Here's a graph, and the associated data: your job is to find **a few** of the things needed for the calculation of Moran's I (you do not need to compute Moran's I).

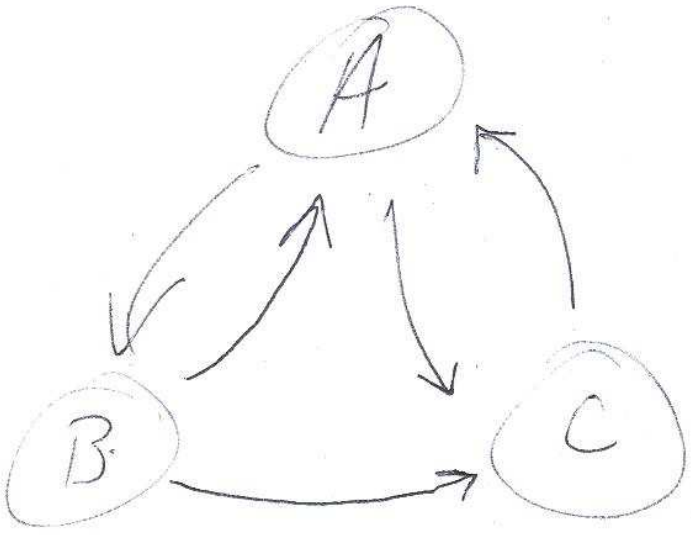


a. What is the adjacency matrix (assuming rook's adjacency)? Draw it next to the graph above.

b. What is the data vector?

c. Set up (but do not calculate) the matrix/data vector product that I've called "the big ugly": $\sum_i \sum_j w_{ij} \left(X_i - \overline{X}\right) \left(X_j - \overline{X}\right)$.

Problem 3: (10 pts) Suppose that the following graph represents a small web.



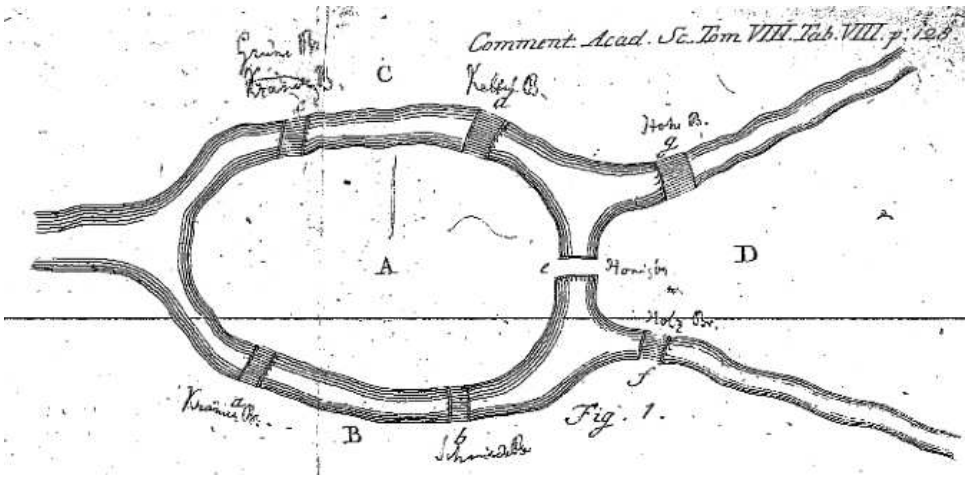
We want to use Google's PageRank algorithm to rate each page.

a. Write the dynamic equations for updating each page's rank.

b. Starting from initial page values of 1/3 each, compute the next two sets of values for the three pages, A, B, and C, and add them to this table:

$$\begin{bmatrix} n & A & B & C \\ 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 1 & & & \\ 2 & & & \end{bmatrix}$$

Problem 4: (10 pts) Below is the situation Euler considered when he single-handedly invented graph theory. How did Euler ruin the fun of the citizens of Konigsberg?



In particular,

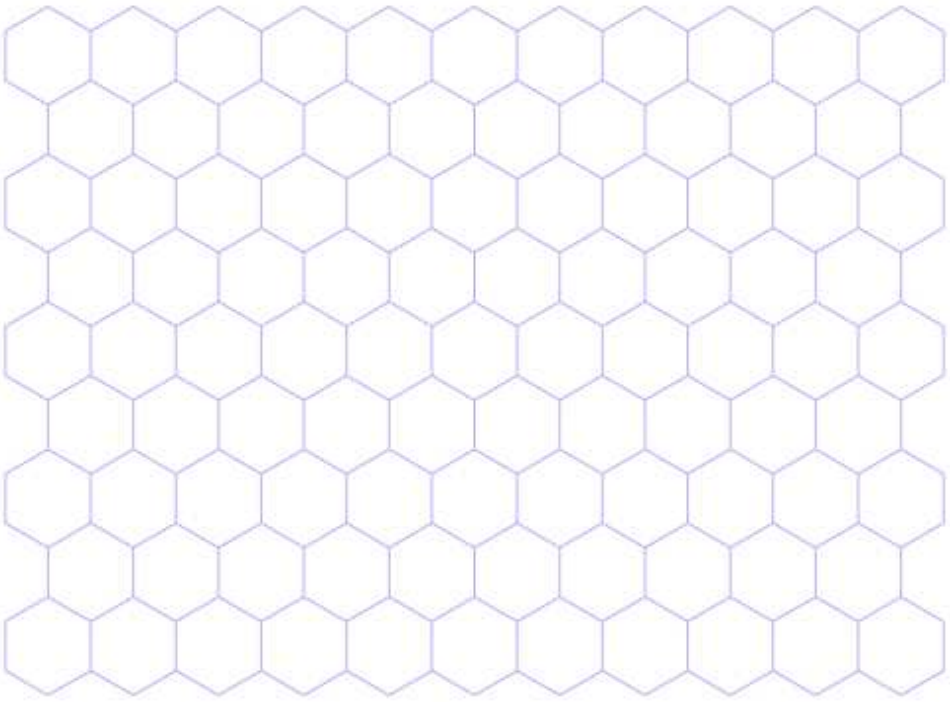
a. (4 pts) Turn this image into a graph, explaining Euler's essential ideas.

b. (4 pts) Explain what Euler determined as essential for the existence of an "Euler path", and how he knew that this graph didn't have one.

c. (2 pts) Is this graph planar?

Problem 5: (20 pts)

a. (10 pts) Use the hexagonal grid to create Pascal's triangle:



b. (5 pts)

i. Describe the relationship between Pascal's triangle and counting simple, **labelled** graphs.

ii. For our purposes here, consider those with four labelled vertices: which row in Pascal's triangle informs us about numbers of different kinds of four-vertex labelled graphs, and what does it tell us?

c. (5 pts) Draw all distinctly different **unlabelled** simple graphs with four vertices that have three edges. Which are dual to each other? Which are self-dual?