MAT115H: soil lead profile for the Newport Foundry site, Newport, KY
Fall, 2015
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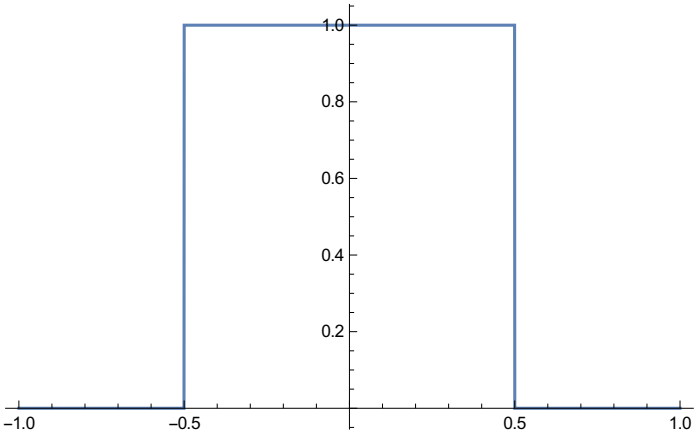
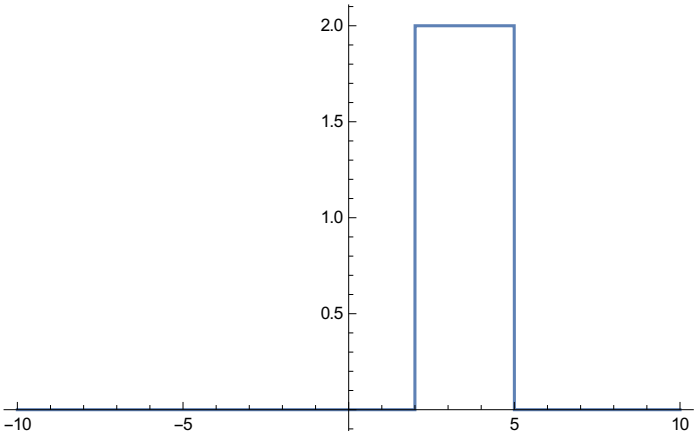
- Data is provided by the report of the KY EPA

In[1]:=
```
(* I'm using UnitBox to create my step-functions: *)
Plot[UnitBox[x], {x, -1, 1}]
(* I'll define mine as LeadStep: *)
LeadStep[x_, a_, b_, c_] := c * UnitBox[(x - a) / (b - a) - .5]
Plot[LeadStep[x, 2, 5, 2], {x, -10, 10}]
```

Out[1]=



Out[3]=

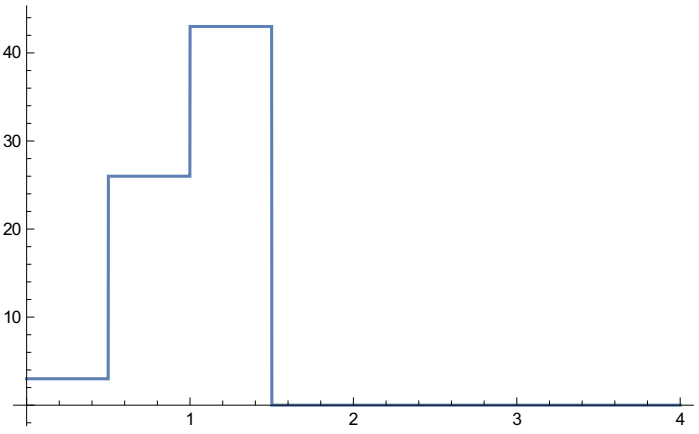


Now for the data:

In[4]:=
```
LH01a[x_] := LeadStep[x, 0, .5, 3]
LH01b[x_] := LeadStep[x, .5, 1, 26]
LH01c[x_] := LeadStep[x, 1, 1.5, 43]
LH01[x_] := LH01a[x] + LH01b[x] + LH01c[x]
LH01[4]
Plot[LH01[x], {x, 0, 4}]
```

Out[8]= 0

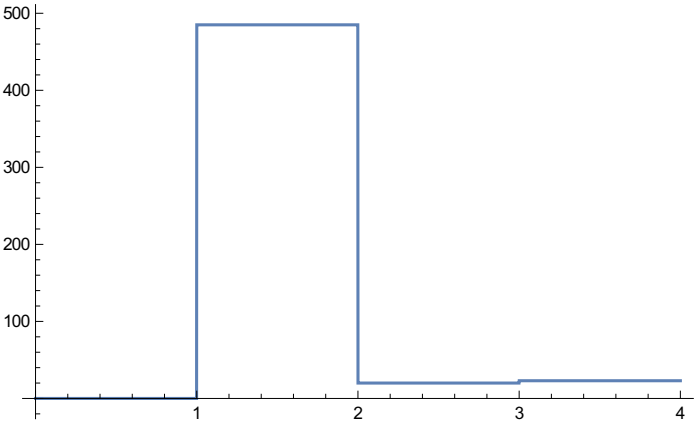
Out[9]=



In[10]:=

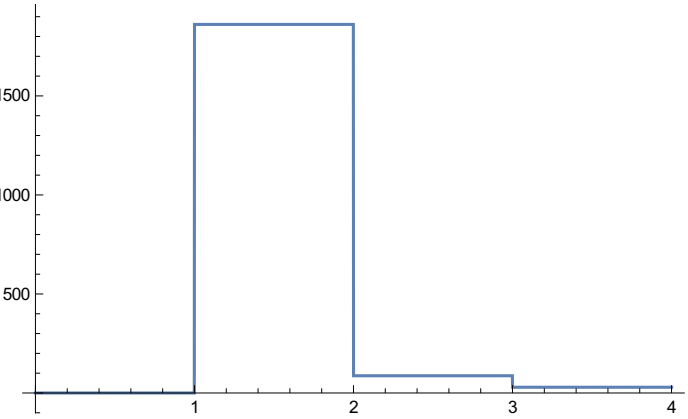
In[11]:=
```
LH02a[x_] := LeadStep[x, 1, 2, 485]
LH02b[x_] := LeadStep[x, 2, 3, 20]
LH02c[x_] := LeadStep[x, 3, 4, 23]
LH02[x_] := LH02a[x] + LH02b[x] + LH02c[x]
Plot[LH02[x], {x, 0, 4}]
```

Out[15]=



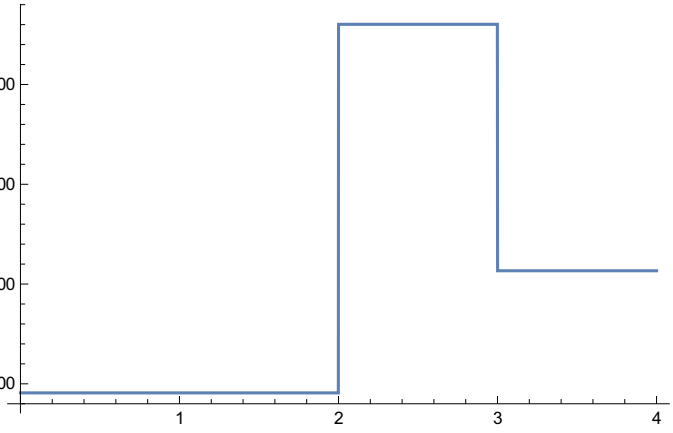
In[16]:=
```
LH03a[x_] := LeadStep[x, 1, 2, 1861]
LH03b[x_] := LeadStep[x, 2, 3, 87]
LH03c[x_] := LeadStep[x, 3, 4, 29]
LH03[x_] := LH03a[x] + LH03b[x] + LH03c[x]
Plot[LH03[x], {x, 0, 4}]
```

Out[20]=



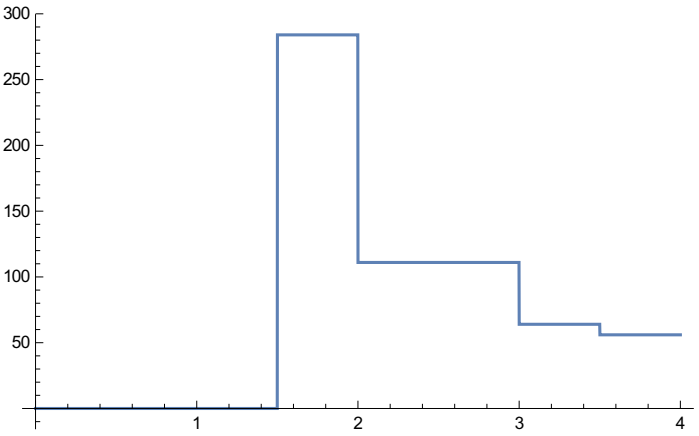
In[21]:=
```
LH04a[x_] := LeadStep[x, 0, 2, 909]
LH04b[x_] := LeadStep[x, 2, 3, 4603]
LH04c[x_] := LeadStep[x, 3, 4, 2133]
LH04[x_] := LH04a[x] + LH04b[x] + LH04c[x]
Plot[LH04[x], {x, 0, 4}]
```

Out[25]=



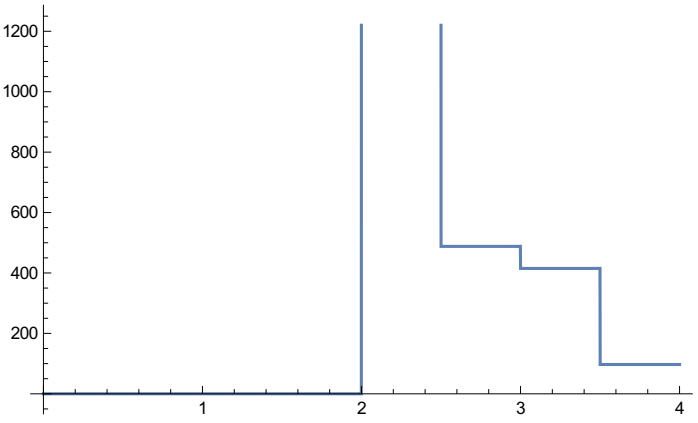
In[26]:=
```
LH05a[x_] := LeadStep[x, 1.5, 2, 284]
LH05b[x_] := LeadStep[x, 2, 3, 111]
LH05c[x_] := LeadStep[x, 3, 3.5, 64]
LH05d[x_] := LeadStep[x, 3.5, 4, 56]
LH05[x_] := LH05a[x] + LH05b[x] + LH05c[x] + LH05d[x]
Plot[LH05[x], {x, 0, 4}]
```

Out[31]=



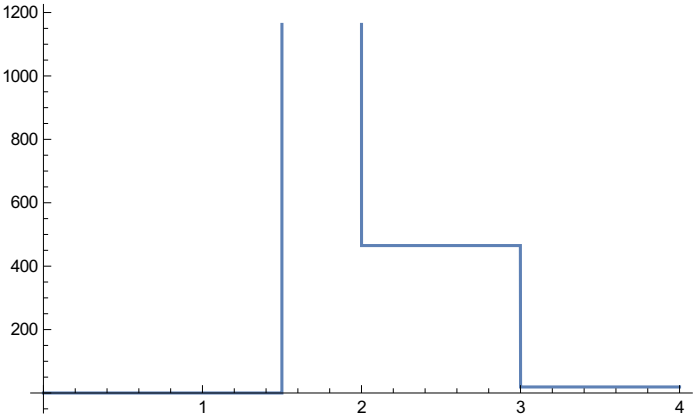
In[32]:=
```
LH06a[x_] := LeadStep[x, 2, 2.5, 4660]
LH06b[x_] := LeadStep[x, 2.5, 3, 488]
LH06c[x_] := LeadStep[x, 3, 3.5, 415]
LH06d[x_] := LeadStep[x, 3.5, 4, 97]
LH06[x_] := LH06a[x] + LH06b[x] + LH06c[x] + LH06d[x]
Plot[LH06[x], {x, 0, 4}]
```

Out[37]=



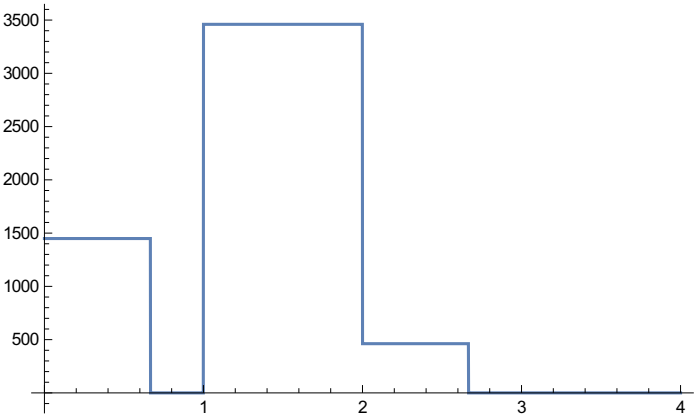
In[38]:=
```
LH07a[x_] := LeadStep[x, 1.5, 2, 5347]
LH07b[x_] := LeadStep[x, 2, 3, 465]
LH07c[x_] := LeadStep[x, 3, 4, 19]
LH07[x_] := LH07a[x] + LH07b[x] + LH07c[x]
Plot[LH07[x], {x, 0, 4}]
```

Out[42]=



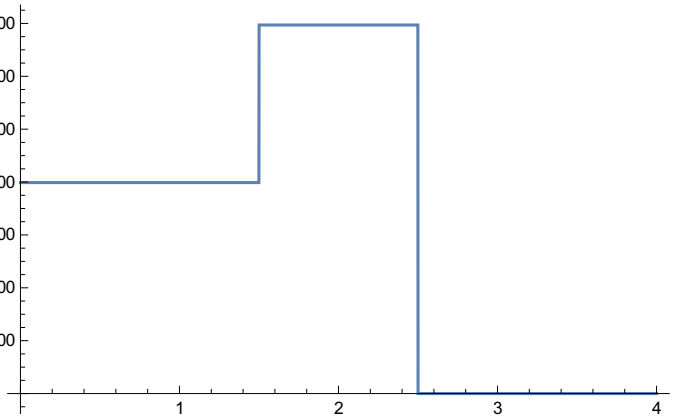
In[43]:=
```
LH08a[x_] := LeadStep[x, 0, 2 / 3, 1449]
LH08b[x_] := LeadStep[x, 1, 2, 3460]
LH08c[x_] := LeadStep[x, 2, 8 / 3, 462]
LH08[x_] := LH08a[x] + LH08b[x] + LH08c[x]
Plot[LH08[x], {x, 0, 4}]
```

Out[47]=



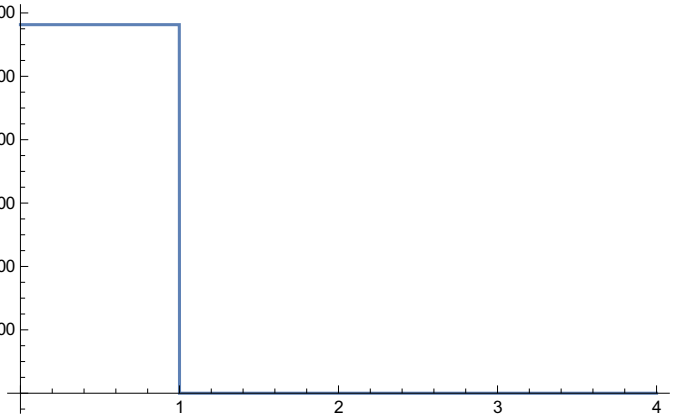
In[48]:=
```
LH09a[x_] := LeadStep[x, 0, 1.5, 798]
LH09b[x_] := LeadStep[x, 1.5, 2.5, 1394]
LH09[x_] := LH09a[x] + LH09b[x]
Plot[LH09[x], {x, 0, 4}]
```

Out[51]=



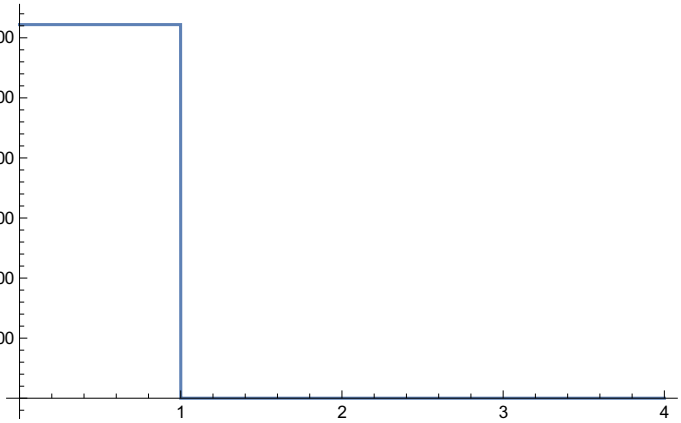
In[52]:=
```
LH10[x_] := LeadStep[x, 0, 1, 1163]
Plot[LH10[x], {x, 0, 4}]
```

Out[53]=



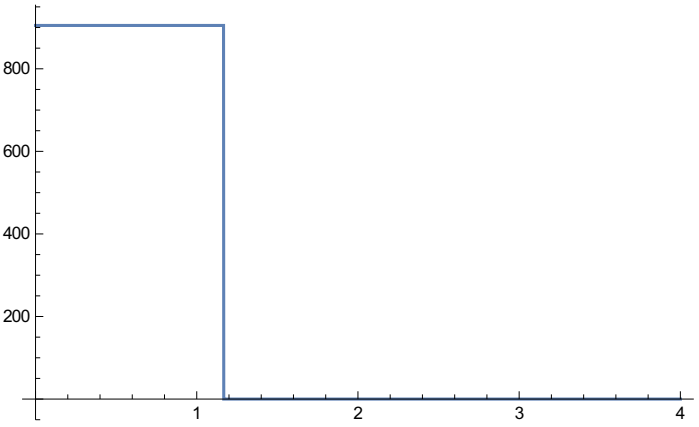
In[54]:=
```
LH11[x_] := LeadStep[x, 0, 1, 622]
Plot[LH11[x], {x, 0, 4}]
```

Out[55]=



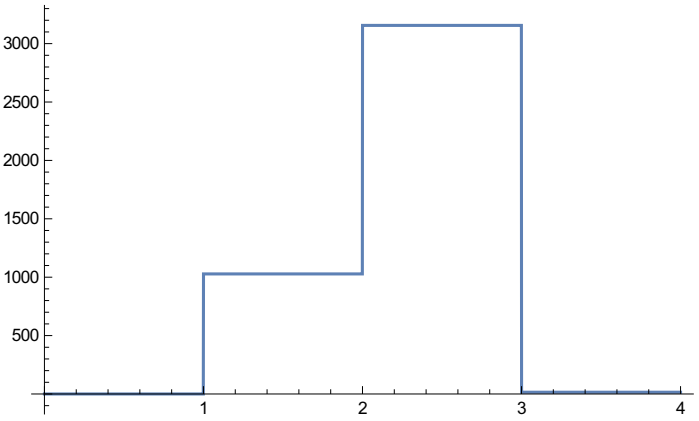
In[56]:=
```
LH12[x_] := LeadStep[x, 0, 7 / 6, 905]
Plot[LH12[x], {x, 0, 4}]
```

Out[57]=



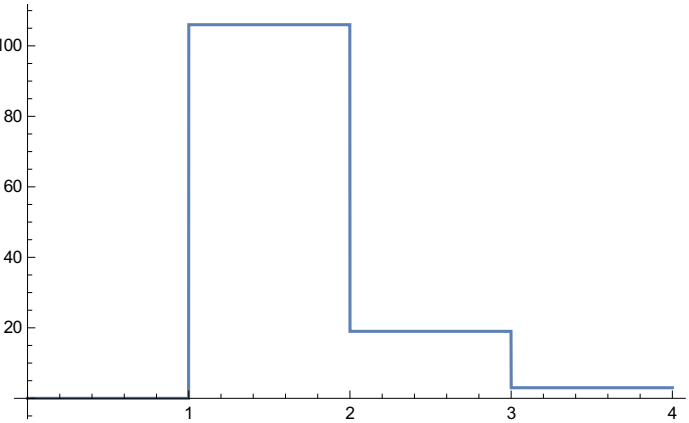
In[58]:=
```
LH13a[x_] := LeadStep[x, 1, 2, 1028]
LH13b[x_] := LeadStep[x, 2, 3, 3156]
LH13c[x_] := LeadStep[x, 3, 4, 15]
LH13[x_] := LH13a[x] + LH13b[x] + LH13c[x]
Plot[LH13[x], {x, 0, 4}]
```

Out[62]=



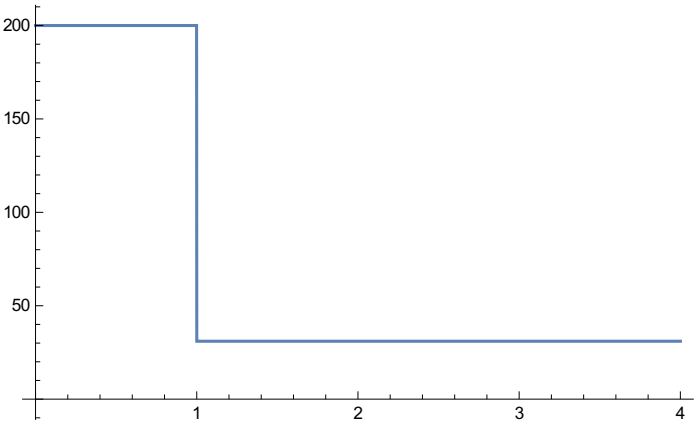
In[63]:=
```
LH14a[x_] := LeadStep[x, 1, 2, 106]
LH14b[x_] := LeadStep[x, 2, 3, 19]
LH14c[x_] := LeadStep[x, 3, 4, 3]
LH14[x_] := LH14a[x] + LH14b[x] + LH14c[x]
Plot[LH14[x], {x, 0, 4}]
```

Out[67]=



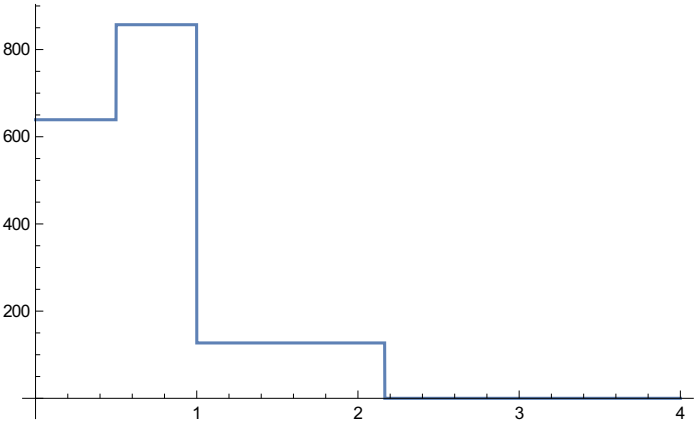
In[68]:=
```
LH15a[x_] := LeadStep[x, 0, 1, 200]
LH15b[x_] := LeadStep[x, 1, 4, 31]
LH15[x_] := LH15a[x] + LH15b[x]
Plot[LH15[x], {x, 0, 4}]
```

Out[71]=



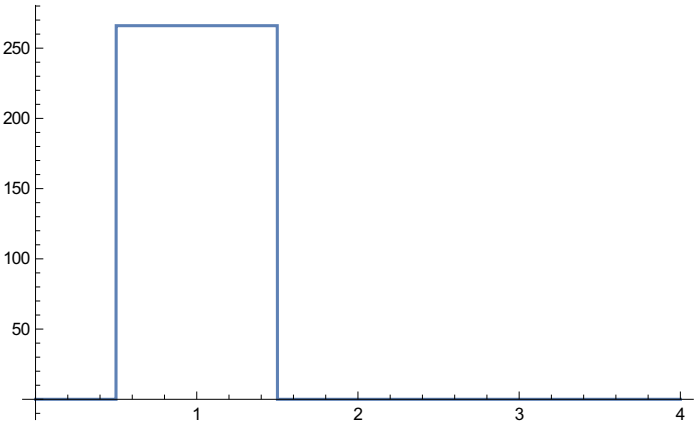
In[72]:=
```
LH16a[x_] := LeadStep[x, 0, .5, 639]
LH16b[x_] := LeadStep[x, .5, 1, 857]
LH16c[x_] := LeadStep[x, 1, 13 / 6, 127]
LH16[x_] := LH16a[x] + LH16b[x] + LH16c[x]
Plot[LH16[x], {x, 0, 4}]
```

Out[76]=



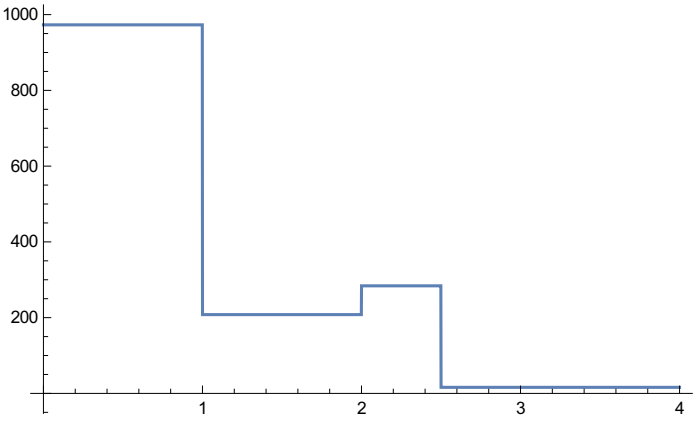
In[77]:=
```
LH17[x_] := LeadStep[x, 0.5, 1.5, 266]
Plot[LH17[x], {x, 0, 4}]
```

Out[78]=



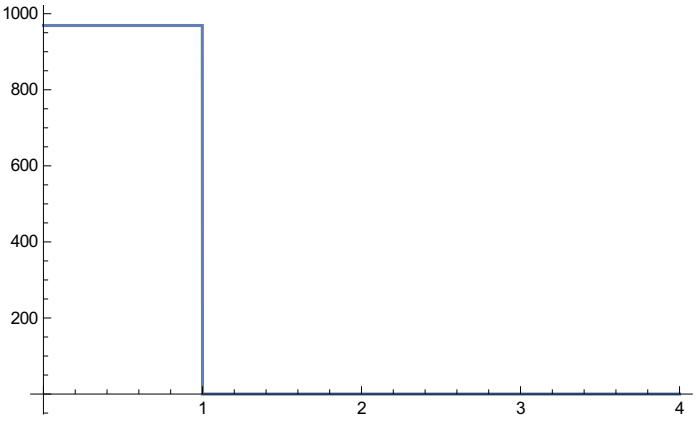
In[79]:=
```
LH18a[x_] := LeadStep[x, 0, 1, 973]
LH18b[x_] := LeadStep[x, 1, 2, 208]
LH18c[x_] := LeadStep[x, 2, 2.5, 284]
LH18d[x_] := LeadStep[x, 2.5, 4, 16]
LH18[x_] := LH18a[x] + LH18b[x] + LH18c[x] + LH18d[x]
Plot[LH18[x], {x, 0, 4}]
```

Out[84]=



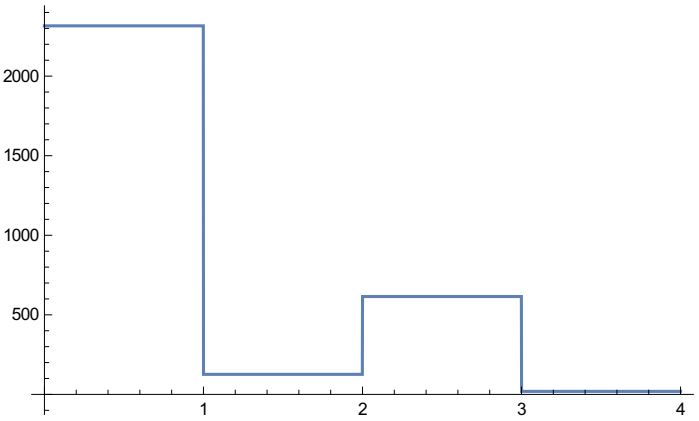
In[85]:=
```
LH19[x_] := LeadStep[x, 0, 1, 969]
Plot[LH19[x], {x, 0, 4}]
```

Out[86]=



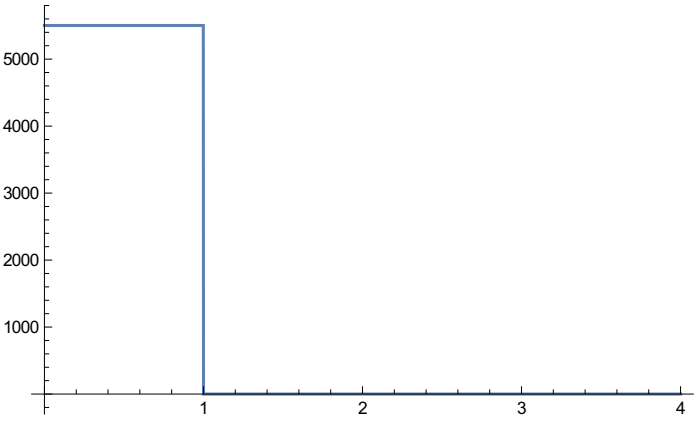
In[87]:=
```
LH20a[x_] := LeadStep[x, 0, 1, 2316]
LH20b[x_] := LeadStep[x, 1, 2, 126]
LH20c[x_] := LeadStep[x, 2, 3, 615]
LH20d[x_] := LeadStep[x, 3, 4, 19]
LH20[x_] := LH20a[x] + LH20b[x] + LH20c[x] + LH20d[x]
Plot[LH20[x], {x, 0, 4}]
```

Out[92]=



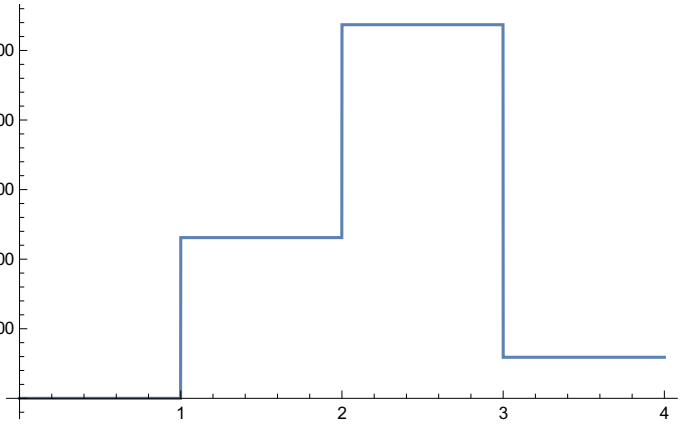
In[93]:=
```
LH21[x_] := LeadStep[x, 0, 1, 5502]
Plot[LH21[x], {x, 0, 4}]
```

Out[94]=



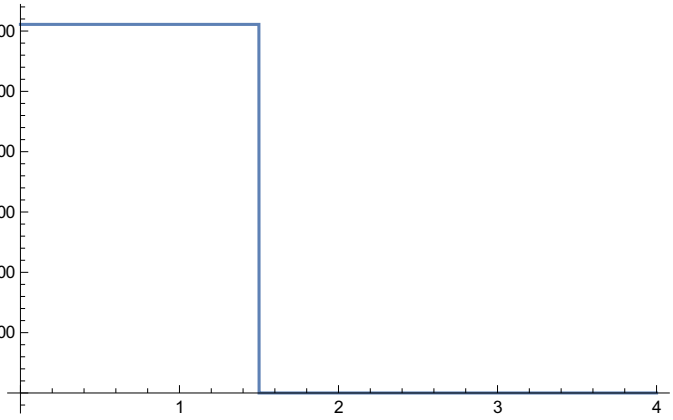
In[95]:=
```
LH22a[x_] := LeadStep[x, 1, 2, 231]
LH22b[x_] := LeadStep[x, 2, 3, 537]
LH22c[x_] := LeadStep[x, 3, 4, 59]
LH22[x_] := LH22a[x] + LH22b[x] + LH22c[x]
Plot[LH22[x], {x, 0, 4}]
```

Out[99]=



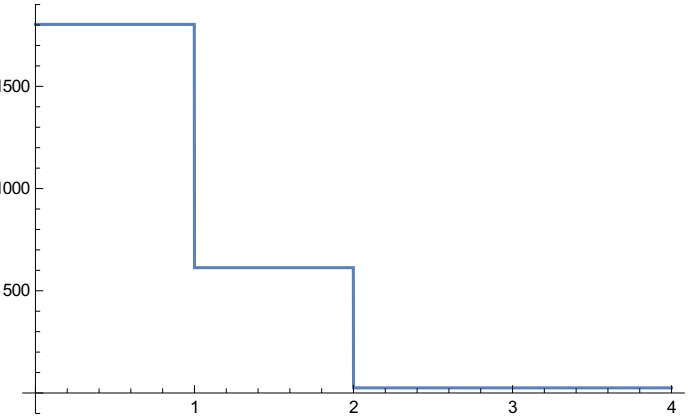
In[100]:=
```
LH23[x_] := LeadStep[x, 0, 1.5, 3055]
Plot[LH23[x], {x, 0, 4}]
```

Out[101]=



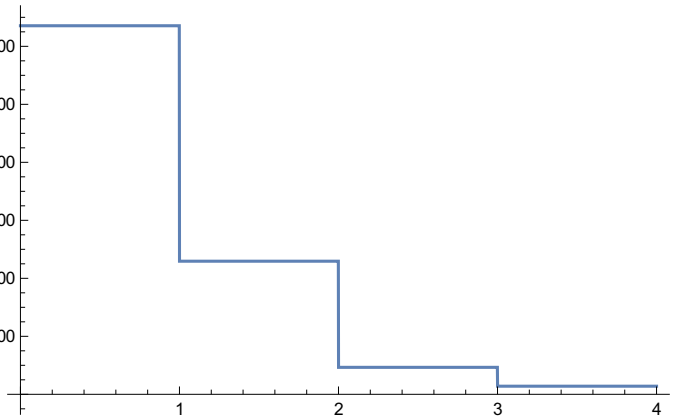
In[102]:=
```
LH24a[x_] := LeadStep[x, 0, 1, 1803]
LH24b[x_] := LeadStep[x, 1, 2, 613]
LH24c[x_] := LeadStep[x, 2, 4, 25]
LH24[x_] := LH24a[x] + LH24b[x] + LH24c[x]
Plot[LH24[x], {x, 0, 4}]
```

Out[106]=



In[107]:=
```
LH25a[x_] := LeadStep[x, 0, 1, 1271]
LH25b[x_] := LeadStep[x, 1, 2, 459]
LH25c[x_] := LeadStep[x, 2, 3, 93]
LH25d[x_] := LeadStep[x, 3, 4, 28]
LH25[x_] := LH25a[x] + LH25b[x] + LH25c[x] + LH25d[x]
Plot[LH25[x], {x, 0, 4}]
```

Out[112]=

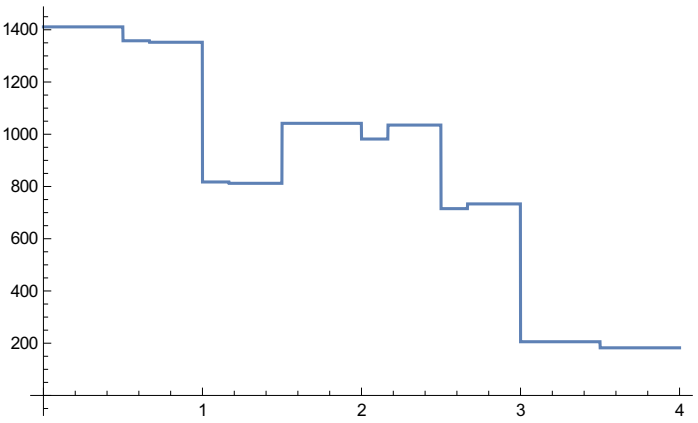


- Here, then, is our data, combined together. I include a “one standard-deviation band” around the data. This is a combination of all of those step functions which make up the set of 25 samples, with various numbers of samples taken in a soil column.

In[113]:=
```
SoilLeadProfile[x_] :=
 Module[{cases =
    DeleteCases[{
      LH01[x], LH02[x], LH03[x], LH04[x], LH05[x], LH06[x], LH07[x], LH08[x], LH09[x],
      LH10[x], LH11[x], LH12[x], LH13[x], LH14[x], LH15[x], LH16[x], LH17[x],
      LH18[x], LH19[x], LH20[x], LH21[x], LH22[x], LH23[x], LH24[x], LH25[x]
     }, 0]},
  (* Print[{x,cases}]; *)
  Mean[cases]
 ]
SoilLeadProfileSD[x_] :=
 Module[{cases =
    DeleteCases[{
      LH01[x], LH02[x], LH03[x], LH04[x], LH05[x], LH06[x], LH07[x], LH08[x], LH09[x],
      LH10[x], LH11[x], LH12[x], LH13[x], LH14[x], LH15[x], LH16[x], LH17[x],
      LH18[x], LH19[x], LH20[x], LH21[x], LH22[x], LH23[x], LH24[x], LH25[x]
     }, 0]},
  StandardDeviation[cases]
 ]
```

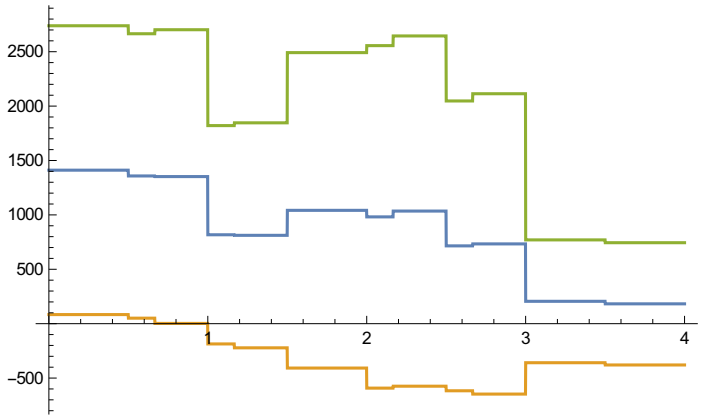
In[115]:=
```
Plot[SoilLeadProfile[x], {x, 0, 4}]
```

Out[115]=



In[116]:=
```
Plot[{SoilLeadProfile[x],
  SoilLeadProfile[x] - SoilLeadProfileSD[x],
  SoilLeadProfile[x] + SoilLeadProfileSD[x]},
 {x, 0, 4}]
```

Out[116]=



- Frequently data like these lead values needs to be log-transformed, to get a data set that looks slightly more normally distributed:

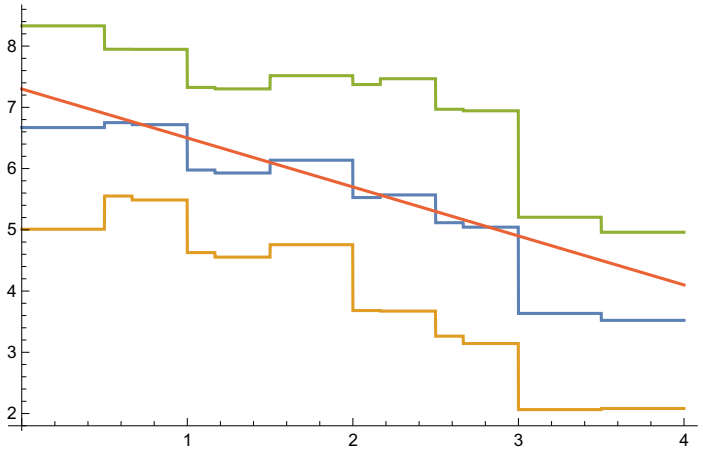
In[117]:=
```
SoilLogLeadProfile[x_] :=
 Module[{cases =
    DeleteCases[{
      LH01[x], LH02[x], LH03[x], LH04[x], LH05[x], LH06[x], LH07[x], LH08[x], LH09[x],
      LH10[x], LH11[x], LH12[x], LH13[x], LH14[x], LH15[x], LH16[x], LH17[x],
      LH18[x], LH19[x], LH20[x], LH21[x], LH22[x], LH23[x], LH24[x], LH25[x]
     }, 0]},
  Mean[Log[cases]]
 ]
SoilLogLeadProfileSD[x_] :=
 Module[{cases =
    DeleteCases[{
      LH01[x], LH02[x], LH03[x], LH04[x], LH05[x], LH06[x], LH07[x], LH08[x], LH09[x],
      LH10[x], LH11[x], LH12[x], LH13[x], LH14[x], LH15[x], LH16[x], LH17[x],
      LH18[x], LH19[x], LH20[x], LH21[x], LH22[x], LH23[x], LH24[x], LH25[x]
     }, 0]},
  StandardDeviation[Log[cases]]
 ]
```

In[119]:=
```
m = -3.2 / 4
b = 7.3
model[x_] := m x + b
Plot[{SoilLogLeadProfile[x],
  SoilLogLeadProfile[x] - SoilLogLeadProfileSD[x],
  SoilLogLeadProfile[x] + SoilLogLeadProfileSD[x],
  model[x]},
 {x, 0, 4}]
Plot[{SoilLeadProfile[x], E^model[x]}, {x, 0, 4}]
```

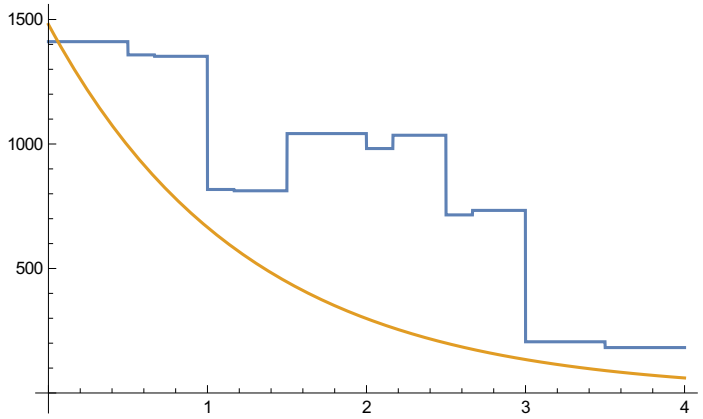
Out[119]= -0.8

Out[120]= 7.3

Out[122]=



Out[123]=



In[124]:=
```
SoilLogLeadProfile[x]
N[SoilLogLeadProfile[.3]]
```

Out[124]=
$$\frac{1}{25}\left(\text{Log}[97\,\text{UnitBox}[0.5-2.\,(-3.5+x)]+415\,\text{UnitBox}[0.5-2.\,(-3+x)]+488\,\text{UnitBox}[0.5-2.\,(-2.5+x)]+4660\,\text{UnitBox}[0.5-2.\,(-2+x)]]+\text{Log}[266\,\text{UnitBox}[0.5-1.\,(-0.5+x)]]+\text{Log}[43\,\text{UnitBox}[0.5-2.\,(-1+x)]+26\,\text{UnitBox}[0.5-2.\,(-0.5+x)]+3\,\text{UnitBox}[0.5-2.\,x]]+\text{Log}\left[127\,\text{UnitBox}\left[0.5-\frac{6}{7}\,(-1+x)\right]+857\,\text{UnitBox}[0.5-2.\,(-0.5+x)]+639\,\text{UnitBox}[0.5-2.\,x]\right]+\text{Log}[622\,\text{UnitBox}[0.5-x]]+\text{Log}[969\,\text{UnitBox}[0.5-x]]+\text{Log}[1163\,\text{UnitBox}[0.5-x]]+\text{Log}[5502\,\text{UnitBox}[0.5-x]]+\text{Log}\left[31\,\text{UnitBox}\left[0.5+\frac{1-x}{3}\right]+200\,\text{UnitBox}[0.5-x]\right]+\text{Log}[16\,\text{UnitBox}[0.5-0.666667\,(-2.5+x)]+284\,\text{UnitBox}[0.5-2.\,(-2+x)]+973\,\text{UnitBox}[0.5-x]+208\,\text{UnitBox}[1.5-x]]+\text{Log}\left[25\,\text{UnitBox}\left[0.5+\frac{2-x}{2}\right]+1803\,\text{UnitBox}[0.5-x]+613\,\text{UnitBox}[1.5-x]\right]+\text{Log}\left[462\,\text{UnitBox}\left[0.5-\frac{3}{2}\,(-2+x)\right]+1449\,\text{UnitBox}\left[0.5-\frac{3x}{2}\right]+3460\,\text{UnitBox}[1.5-x]\right]+\text{Log}[56\,\text{UnitBox}[0.5-2.\,(-3.5+x)]+64\,\text{UnitBox}[0.5-2.\,(-3+x)]+284\,\text{UnitBox}[0.5-2.\,(-1.5+x)]+111\,\text{UnitBox}[2.5-x]]+\text{Log}[106\,\text{UnitBox}[1.5-x]+19\,\text{UnitBox}[2.5-x]+3\,\text{UnitBox}[3.5-x]]+\text{Log}[1028\,\text{UnitBox}[1.5-x]+3156\,\text{UnitBox}[2.5-x]+15\,\text{UnitBox}[3.5-x]]+\text{Log}[5347\,\text{UnitBox}[0.5-2.\,(-1.5+x)]+465\,\text{UnitBox}[2.5-x]+19\,\text{UnitBox}[3.5-x]]+\text{Log}[2316\,\text{UnitBox}[0.5-x]+126\,\text{UnitBox}[1.5-x]+615\,\text{UnitBox}[2.5-x]+19\,\text{UnitBox}[3.5-x]]+\text{Log}[485\,\text{UnitBox}[1.5-x]+20\,\text{UnitBox}[2.5-x]+23\,\text{UnitBox}[3.5-x]]+\text{Log}[1271\,\text{UnitBox}[0.5-x]+459\,\text{UnitBox}[1.5-x]+93\,\text{UnitBox}[2.5-x]+28\,\text{UnitBox}[3.5-x]]+\text{Log}[1861\,\text{UnitBox}[1.5-x]+87\,\text{UnitBox}[2.5-x]+29\,\text{UnitBox}[3.5-x]]+\text{Log}[231\,\text{UnitBox}[1.5-x]+537\,\text{UnitBox}[2.5-x]+59\,\text{UnitBox}[3.5-x]]+\text{Log}\left[905\,\text{UnitBox}\left[0.5-\frac{6x}{7}\right]\right]+\text{Log}[3055\,\text{UnitBox}[0.5-0.666667\,x]]+\text{Log}[1394\,\text{UnitBox}[0.5-1.\,(-1.5+x)]+798\,\text{UnitBox}[0.5-0.666667\,x]]+\text{Log}\left[4603\,\text{UnitBox}[2.5-x]+2133\,\text{UnitBox}[3.5-x]+909\,\text{UnitBox}\left[0.5-\frac{x}{2}\right]\right]\right)$$

Out[125]=
```
6.66975
```

In[126]:=
```
model[a_, b_] := NIntegrate[(N[SoilLeadProfile[x]] - E^(a x + b))^2, {x, 0, 4}]
model[-.5, 7]
```

Out[127]=
```
101 414.
```

In[128]:=
```
(* nlm=NonlinearModelFit[data,a Log[b x +c y],{a,b,c},{x,y},Weights→1/errors^2] *)
```

In[129]:=
```
nlm = NMinimize[{model[a, b], a < 0}, {a, b} ∈ 10 * Disk[]]
```

NIntegrate::inumr : The integrand
$\left(-e^{7.3+a\,x} + 0.04\,(31.\,\text{UnitBox}[0.5 + \text{Times}[\ll 2\gg]] + 25.\,\text{UnitBox}[0.5 + \text{Times}[\ll 2\gg]] + \ll 20\gg + 909.\,\text{UnitBox}[0.5 + \text{Times}[\ll 2\gg]])\right)^2$ has evaluated to non−numerical values for all sampling points in the region with boundaries {{0, 4}}. ≫

NMinimize::ivar : {a, 7.3} ∈ 10 Disk[{0, 0}] is not a valid variable. ≫

NIntegrate::inumr : The integrand
$\left(-e^{7.3+a\,x} + 0.04\,(31.\,\text{UnitBox}[0.5 + \text{Times}[\ll 2\gg]] + 25.\,\text{UnitBox}[0.5 + \text{Times}[\ll 2\gg]] + \ll 20\gg + 909.\,\text{UnitBox}[0.5 + \text{Times}[\ll 2\gg]])\right)^2$ has evaluated to non−numerical values for all sampling points in the region with boundaries {{0, 4}}. ≫

NMinimize::ivar : {a, 7.3} ∈ 10 Disk[{0, 0}] is not a valid variable. ≫

Out[129]=
```
NMinimize[{NIntegrate[(N[SoilLeadProfile[x]] - e^(a x+7.3))^2, {x, 0, 4}], a < 0},
 {a, 7.3} ∈ 10 Disk[{0, 0}]]
```