The experiment is done. You have completed all your measurements and recorded all your trials as Raw Data. What’s next? Any experimental result only has meaning if it is appropriately analyzed. The four steps given below explain how to get a meaningful experimental result from recorded measurements. Three of these steps are completed as Data Analysis. The final (and perhaps the most important step) is completed as the Assessment and Conclusion. Note: On occasion, Data Analysis Steps 2 and 3 below can be replaced by plotting and fitting data (see Appendix B in manual).

Data Analysis:

**Step #1:** Find the mean value of each measured quantity and the total uncertainty (TU) associated with each mean.

Measurements are repeated to calculate a mean value ($X_m$). The total uncertainty (TU) of the mean is found by combining instrumental uncertainty (IU) and random uncertainty (RU).

- **IU** = one half the smallest division on the instrument’s scale OR one half the last recorded significant decimal place.
- **RU** = $\frac{s}{\sqrt{N}}$ where $s$ = standard deviation of the sample of data for this measurement and $N$ = number of trials. (If repeated trials were not done, then ask instructor for an estimated value for RU.) If you are not sure how to calculate $s$ (sample standard deviation), then read Calculating the Standard Deviation of a Sample in Appendix A.
- Combine IU and RU to find the total uncertainty in your mean measurement:

$$TU = \sqrt{(IU)^2 + (RU)^2}.$$ Now record the mean measurement as $X_m \pm TU$.

**Step #2:** Calculate the final experimental result as a function ($f$) of one or more mean measurements. Find the total uncertainty of the final result (TU) by propagating the uncertainty of the mean measurements into the final result.

Let $x$ and $y$ be mean measurements and $TU_x$ and $TU_y$ be their respective total uncertainties. Let $a$, $b$ and the powers $m$ and $n$ be known constants. Propagation of uncertainty is done as follows:

- If $f$ is found by adding/subtracting only: $f = ax \pm by$ then $TU_f = \sqrt{a^2(TU_x)^2 + b^2(TU_y)^2}$
- If $f$ is found by multiplication/division only: $f = ax^n y^m$ then $TU_f = f \sqrt{\left(\frac{m(TU_y)}{y}\right)^2 + \left(\frac{n(TU_x)}{x}\right)^2}$

If the final result is a function ($f$) of three mean measurements, then just add a $3^{rd}$ term under the radicals above; for example, if $f = ax \pm by \pm cz$, then add $c^2(TU_z)^2$ under the radical. Likewise, if $f = ax^n y^m z^p$ then add the fraction $\left(\frac{p(TU_z)}{z}\right)^2$ under the radical.

If the final result is a combination of adding/subtracting and multiplying/dividing, then use steps to solve. For example, suppose $f = (ax \pm by)(ax^n y^m)$. Let $f_1 = ax \pm by$ and $f_2 = ax^n y^m$ then find the TU values for $f_1$ and $f_2$. Now use the multiplication approach above to get $TU$ for $f$ where $f = f_1 f_2$ with $f_1$ and $f_2$ being to the 1" power.
Step #3: Compare the final result to a “true” value OR to the same result obtained by other methods. Compare the amount of error (or difference) to what is allowed by total uncertainty.

First find the percentage amount of uncertainty (or “fuzziness”) in your result:

\[ \% TU_f = 100\% \cdot \left( \frac{TU_f}{f} \right) \]

Then pick the appropriate path below:

Path A: Your final result \((f \pm TU_f)\) is compared to a true (or accepted) value \((f_{true})\).
- Find the percent error (or deviation from true) of your result:
  \[ \% \text{ error} = 100\% \cdot \left( \frac{f - f_{true}}{f_{true}} \right) \]
- Determine if the experimental error has been successfully minimized:
  If \(|% \text{ error}| < |%TU_f|\), then error is less than the allowed uncertainty (or “fuzziness”) and error has been successfully minimized.
  If \(|% \text{ error}| > |%TU_f|\), but \(|% \text{ error}| < 2 |%TU_f|\) then a small amount of error exists and/or the uncertainty was underestimated.
  If \(|% \text{ error}| > 2 |%TU_f|\) then a significant amount of error exists, and either mistakes were made and/or the uncertainty was underestimated.

Path B: If there is no true (or accepted) value available, then your final result \((f \pm TU_f)\) is compared to the result obtained by another method or experiment \((f_{other} \pm TU_{other})\).
- Find the average result:
  \[ f_{ave} = \frac{(f + f_{other})}{2} \]
  and then find the combined total uncertainty of the two methods (or experiments) and the percent difference:
  \[ % TU_{combo} = \sqrt{\left(\frac{\%TU_f}{f_{ave}}\right)^2 + \left(\frac{\%TU_{other}}{f_{ave}}\right)^2} \]
  \[ \% \text{ diff} = 100\% \cdot \left( \frac{f - f_{other}}{f_{ave}} \right) \]
- Determine whether or not the two methods (or experiments) are consistent:
  If \(|% \text{ diff}| < |%TU_{combo}|\) then disagreement between the two methods is less than the allowed uncertainty (or “fuzziness”) so consistency is good.
  If \(|% \text{ diff}| > |%TU_{combo}|\), but \(|% \text{ diff}| < 2 |%TU_{combo}|\) then a small amount of disagreement exists and/or the uncertainty was underestimated.
  If \(|% \text{ diff}| > 2 |%TU_{combo}|\) then a significant amount of disagreement exists, and either mistakes were made and/or the uncertainty was underestimated.

Assessment and Conclusion:

Step 4: Complete an assessment and reach a conclusion. Indicate all plausible sources of error and plausible additional sources of uncertainty in the result. If possible, explain the size and direction of change in the result for each source of error. Explain the amount of increase in uncertainty from the additional sources of uncertainty found. Possibly modify the result if errors can be calculated. Suggest improvements. Reach a conclusion as to the success of this experiment.

This step should be completed regardless of the outcome of Step #3. That is, even if the % error (or % diff) is less than the allowed uncertainty, it is necessary to complete this step in a thorough manner. For specific examples on how to complete these 4 steps see manual.