Please show your work. In particular write out the numbers you are adding or multiplying to get your answer, or explicitly write out $C(n,r)$ or $P(n,r)$ before giving the numerical value. Point values are in parentheses.

(5 ea.) 1. Let $S = \{l,m,n,o,p,q,r,s,t,u,v,w\}$, $A = \{l,m,n,o,p\}$, $B = \{n,o,p,q,r,s\}$, $C = \{l,m,r,s,t,u,v,w\}$. List the elements in each of the following sets:

a) $A \cap C = \{l,m\}$

b) $A \cup B = \{l,m,n,o,p,q,r,s\}$

c) $A' \cap C = \{q,r,s,t,u,v,w\} \cap \{l,m,r,s,t,u,v,w\} = \{r,s,t,u,v,w\}$

(8) 2. Draw a three set Venn diagram and shade in the set $B \cup C$. Is $A \cap B \cap C'$ a subset of $B \cup C$? Explain.

The region $B \cup C$ is shaded. The small region with the * in it is $A \cap B \cap C'$, which is in the shaded region so $A \cap B \cap C'$ is a subset of $B \cup C$. This can also be justified in several other ways.
(8) 3. In a class with 45 students, 20 are enrolled in a Spanish class, 30 are enrolled in a Music class. However 12 of those in Music are not taking Spanish. Fill out a Venn diagram with this information and use it to tell how many of the students are registered for neither Spanish nor Music.

Let \( A \) be the set of students enrolled in Spanish. \( n(A) = 20 \)
Let \( B \) be the set of students enrolled in Music. \( n(B) = 30 \)
We also have \( n(A' \cap B) = 12 \). Fill in the Venn diagram. From this we see that 13 students are enrolled in neither.

(8) 4. Let \( A \) and \( B \) be any two sets. Explain why \( n(A \cup B) \) is not usually equal to \( n(A) + n(B) \).

The number \( n(A \cup B) \) counts the elements of \( A \cap B \) once, the number \( n(A) + n(B) \) counts the elements of \( A \cap B \) twice. That is why the relationship between them is
\[
n(A \cup B) = n(A) + n(B) - n(A \cap B)
\]

(8) 5. A coffee shop has 12 kinds of bagels, 8 flavors of cream cheese and 17 types of coffee. Pierre wants one bagel, one type of cream cheese on it, and a cup of coffee. How many different options does he face?

He would have \( 12 \cdot 8 \cdot 17 = 1632 \) options.
(8) 6. You have 17 lovely coffee table books. How many different ways are there to arrange 4 of them in a row on a shelf?

In an arrangement order matters so there are \( P(17, 4) = \frac{17!}{13!} = 17 \cdot 16 \cdot 15 \cdot 14 = 57120 \) different arrangements.

(8) 7. A sequence of two different letters is selected from the word *atom* (note: *am* and *ma* are different sequences.) Write out the sample space for this, and then write out the set representing the event “one of the letters is a vowel.”

The sample space is \( S = \{at, ao, am, ta, to, tm, oa, ot, om, ma, mt, mo\} \).

The event is \( \{at, ao, am, ta, to, oa, ot, om, ma, mo\} \)

(5 ea.) 8. A bag contains 5 gum drops, 8 cinnamon candies, and 7 prune candies, all the same size and weight. You reach in and select 4 of them at random, all at once.

a) How many possible selections of candies are there?

This is an unordered selection of 4 from among 5 + 8 + 7 = 20, so there are \( C(20, 4) = \frac{20!}{16!4!} = 4845 \) different selections.

b) How many selections are there where all four are the same type of candy?

This is either 4 gum drops, so \( C(5, 4) = 5 \), or 4 cinnamons, so \( C(8, 4) = 70 \), or 4 prune, so \( C(7, 4) = 35 \). Since these are mutually exclusive, we have 5 + 70 + 35 = 110 selections.

c) How many selections are there with fewer than two prune candies?

This is either 1 prune and 3 of the 8 + 5 = 13 others, which is \( 7 \cdot C(13, 3) = 7 \cdot 286 = 2002 \), or 4 non-prune, which is \( C(13, 4) = 715 \). Again these are mutually exclusive, so there are 2002 + 715 = 2717 possibilities.
(8) 9. A five card poker hand is drawn from a well shuffled ordinary deck. How many different ways are there to get three of one suit and two of another suit? (For example, the A, 5, 6 of clubs and 10 and J of diamonds.)

There are 4 choices for the suit of the three and \( C(13,3) = 286 \) choices of three cards from that suit and 3 choices for the suit of the two and \( C(13,2) = 78 \) choices for the two cards from that suit.

Thus there are \( 4 \cdot 286 \cdot 3 \cdot 78 = 267,696 \) possibilities.

(7 ea.) 10. A group of people were tested for blood sugar levels, rated high, normal or low. The results are in the following table:

<table>
<thead>
<tr>
<th></th>
<th>H</th>
<th>N</th>
<th>L</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Men</td>
<td>16</td>
<td>26</td>
<td>8</td>
<td>50</td>
</tr>
<tr>
<td>W</td>
<td>10</td>
<td>40</td>
<td>10</td>
<td>60</td>
</tr>
<tr>
<td>Total</td>
<td>26</td>
<td>66</td>
<td>18</td>
<td>110</td>
</tr>
</tbody>
</table>

A person from the group is selected at random. The letters outside the box indicate the abbreviations, so that, for example \( M \) is the event “the person selected is a man,” \( N \) is the event, “the person selected has normal blood sugar,” etc. For each of the following events, describe the event in words, and evaluate the probability the event will occur, based on the data in the table.

a) \( W \cup L' \)

This would represent that “the person selected is either a woman or does not have low blood sugar.” Now \( n(W \cup L') = 102 \). The set includes everyone who is not a male with low blood sugar. so \( P(W \cup L') = \frac{102}{110} \approx 0.927 \)

b) \( M \cap H \)

This would represent the event “the person selected is a male with high blood sugar levels.”

Since \( n(M \cap H) = 16 \), we must have \( P(M \cap H) = \frac{16}{110} = 0.145 \).