MAT 129 – 001
Summer 2011
Comprehensive Exam

Calculators may be used. Show all work.

For differentiation problems, show the details of using the chain rule.
For integration problems, show substitutions and all details.

1. By hand, showing all steps, determine \( \lim_{x \to 1} \frac{1 - x^2}{x^2 - 8x + 7} \).

2. By hand, showing all steps, determine \( \lim_{x \to 0} \frac{\sin 5x}{3x} \).

*3. \( F(x) = x^2f(2x) \). Find \( F'(x) \). Your answer will contain \( f \) and \( f' \).

4. \( \tan(x + y) = 2y \). Find \( \frac{dy}{dx} \).

5. A 13-foot ladder is leaning against a vertical wall. A person begins pulling the foot of the ladder away from the building at a rate of 0.5 ft/s. How fast is the top of the ladder sliding down the wall when the foot of the ladder is 5 ft from the wall?

6. Determine the absolute maximum and absolute minimum on the interval \([ -1, 4 ]\) for the function \( y = 3x^4 - 16x^3 + 18x^2 \).

7. Find the local maxima and minima of \( y = x^3 - 3x^2 + 1 \).
8. A piece of wire 10 m long is cut into two pieces. One piece is bent into a square and the other is formed into a circle. How should the wire be cut so that the total area enclosed is a maximum?

9. Evaluate \( \int \sin^{10} x \cos x \, dx \).

10. Evaluate \( \int (x^6 - 3x^2)^3 (x^5 - x) \, dx \).

11. Evaluate \( \int_{0}^{3} \frac{x}{\sqrt{25 - x^2}} \, dx \).

12. Evaluate \( \int_{0}^{\pi/2} \cos x \sin (\sin x) \, dx \).

13. Set up a definite integral to find the area in the region bounded by
\[
y = 8 - 2x \\
y = x + 8 \\
y = 0
\]
You need not do the integration.

14. Set up a definite integral to find the volume generated by revolving the region bounded by \( y = 2\sqrt{x} \) and \( y = x \) about the x-axis.
You need not do the integration.

15. \( f'(x) = \sin x + 2x \). \( f(0) = 5 \). Find \( f(x) \).