MAT 129 – 003
Spring 2011
Test three
Show all work.
For differentiation problems, show the details of using the chain rule.
For integration problems, show substitutions and all details.

1. Determine the absolute maximum and absolute minimum on the interval [-3, 1] for the function \( f(x) = x^4 - 8x^2 + 2 \).

2. Consider the function \( f(x) = x^3 - 3x^2 + 5 \).
   2a. Determine the intervals on which \( f(x) \) is increasing or decreasing.
   2b. Determine local maximum and minimum values.
   2c. Determine intervals of concavity.
   2d. Determine inflection points.

3. A square-based box-shaped shipping crate is designed to have a volume of 16 cubic feet. The material used to make the base costs twice as much per square foot as the material used to make the sides, and the material used to make the top costs half as much as the material used to make the sides. What are the dimensions of the crate that minimize the cost of the materials?

4. Find \( f \). \( f''(x) = \frac{3}{\sqrt{x}} \). \( f''(4) = 7 \) and \( f(4) = 20 \).

5. Determine the following limit:
   \[
   \lim_{x \to \infty} \frac{x + 2}{\sqrt{9x^2 + 1}}
   \]
6. Approximate the integral \( \int_{1}^{4} \sqrt{x+2} \, dx \) using \( n = 6 \) right-endpoint rectangles.

7. Determine \( \int_{1}^{2} \left( 4x - \frac{2}{x^2} \right) \, dx \).

8. \( g(x) = \int_{1}^{x^2} \sqrt{1+t^3} \, dt \). Determine \( g'(x) \).

9. Determine \( \int \frac{x^3}{\sqrt{1-x^4}} \, dx \).

10. Determine \( \int \sec^2 x \sqrt{\tan x} \, dx \).

11. Determine \( \int_{2}^{1} \frac{x}{(x^2+1)^2} \, dx \).