# 1

\[ y = x^3 - 3x^2 \]

\[ y' = 3x^2 - 6x \]

Let \( y' = 3x^2 - 6x = 0 \)

\[ 3x(x-2) = 0 \]

Critical points \( x = 0 \) \( x = 2 \)

-2, 0, 2

Endpoints \( x = -1 \) \( y = -4 \) abs. min.

-3, 0, 1

Endpoints \( x = 4 \) \( y = 16 \) abs. max.

\[ y = x^4 - 4x^3 \]

\[ y' = 4x^3 - 12x^2 \]

Let \( y' = 4x^3 - 12x^2 = 0 \)

\[ 4x^2(x-3) = 0 \]

Critical points \( x = 0 \) \( x = 3 \)

-2, -1, 1

-3, 0, 1, 5

\[ y'' = 12x^2 - 24x \]

Let \( y'' = 12x^2 - 24x = 0 \)

\[ 12x(x-2) = 0 \]

\( x = 0 \) \( x = 2 \)

\[ y'' > 0 \quad y'' = 0 \quad y'' < 0 \quad y'' = 0 \quad y'' > 0 \]

\( \text{min} \) IP \( \text{max} \) IP \( \text{conc} \)
180000 = A = x \ y \\
F = x + 2 \ y \\
F = x + 2 \ \frac{180000}{x} \\
F' = 1 - \frac{360000}{x^2} \\
\text{let } F' = 1 - \frac{360000}{x^2} = 0 \\
x^2 = 360000 \\
x = 600 \\
y = \frac{180000}{600} = 300 \\
F'' = \frac{-720000}{x^3} > 0 \text{ when } x = 600 \\
\text{4} \\
\phi''(x) = x - \cos x \\
\phi'(x) = x^2 - \sin x + C \\
2 = \phi'(0) = 0 - \sin 0 + C \\
\phi'(x^2) = x^2 - \sin x + 2 \\
\phi(x) = x^4 + \cos x + 2x + C \\
n-2 = \phi(0) = 2 + \cos 0 + 0 + C \\
0 = \phi(x) = x^4 + \cos x + 2x - 3
\[ \lim_{n \to \infty} \sum_{k=1}^{n} \frac{1}{n^3} (2 + \frac{k}{n})^2 = \lim_{n \to \infty} \sum_{k=1}^{n} \frac{1}{n} \left( \frac{2 + k/n}{n} \right)^2 = \lim_{n \to \infty} \sum_{k=1}^{n} \frac{1}{n^2} \left( \frac{2 + k/n}{n} \right)^2 \]

\[ \int_{0}^{2} \left( x^2 + 1 \right) \, dx \]

\[ \frac{5}{2} \int_{0}^{2} x^2 \, dx \]

\[ \frac{5}{2} \left[ \frac{x^3}{3} \right]_{0}^{2} = \frac{5}{2} \left( \frac{8}{3} - 0 \right) = \frac{5}{2} \cdot \frac{8}{3} = \frac{20}{3} \]

\[ \frac{5}{2} \left[ x^{3/2} \right]_{2}^{5} = \frac{5}{2} \left( 5^{3/2} - 2^{3/2} \right) = \frac{5}{2} \left( \sqrt{125} - 2\sqrt{2} \right) = \frac{5}{2} \left( 5\sqrt{5} - 2\sqrt{2} \right) \]

\[ \frac{5}{2} \left( \frac{2}{1} + \frac{1}{2} \cdot \frac{3}{2} + \frac{1}{2} \cdot \frac{5}{2} \right) + \frac{5}{2} \left( \frac{2}{1} + \frac{1}{2} \cdot \frac{3}{2} + \frac{1}{2} \cdot \frac{5}{2} \right) + \frac{5}{2} \left( \frac{2}{1} + \frac{1}{2} \cdot \frac{3}{2} + \frac{1}{2} \cdot \frac{5}{2} \right) \]

\[ \frac{5}{2} \left( \frac{2}{1} \right) + \frac{5}{2} \left( \frac{1}{2} \cdot \frac{3}{2} \right) + \frac{5}{2} \left( \frac{1}{2} \cdot \frac{5}{2} \right) \]

\[ \frac{5}{2} \left( \frac{2}{1} \right) + \frac{5}{2} \left( \frac{1}{2} \cdot \frac{3}{2} \right) + \frac{5}{2} \left( \frac{1}{2} \cdot \frac{5}{2} \right) + \frac{5}{2} \left( \frac{2}{1} \right) + \frac{5}{2} \left( \frac{1}{2} \cdot \frac{3}{2} \right) + \frac{5}{2} \left( \frac{1}{2} \cdot \frac{5}{2} \right) 

\[ = \frac{20}{3} + \frac{5}{2} \left( \frac{3}{4} \right) + \frac{5}{2} \left( \frac{25}{8} \right) \]

\[ = \frac{20}{3} + \frac{15}{8} + \frac{25}{8} = \frac{20}{3} + \frac{40}{8} = \frac{20}{3} + 5 = \frac{20}{3} + \frac{15}{3} = \frac{35}{3} \]

\[ = \frac{35}{3} \]
\[ \int x^3 \left( 4x^{3/2} + x^{3/2} \right) \, dx \]

\[ = \left( 4 \frac{x^{5/2}}{5/2} + \frac{x^{7/2}}{7/2} \right) \frac{1}{3} \]

\[ = (8x^{5/2} + \frac{2}{7} x^{7/2}) \frac{1}{3} \]

\[ = \left[ \frac{8}{3} \left( 3x^2 + \frac{2}{7} \right) \right] - \left[ \frac{8}{3} \left( \frac{2}{7} \right)^{1/2} \right] \]

\[ = \frac{162 - 82}{45} = \frac{80}{45} \]

\[ \text{Let } u = x^2 \]

\[ \frac{du}{dx} = 2x \]

\[ \int 12u \cdot u \, du \]

\[ = 12u^2 \cdot \frac{1}{2} \]

\[ = 6u^2 \]

\[ = 6x^4 \]

\[ \text{Using the power rule} \]

\[ \frac{d}{dx} \left( x^2 \right) = 2x \]

\[ \frac{d}{dx} \left( 2x \right) = 2 \]

\[ \frac{d}{dx} \left( 2x^2 \right) = 4x \]

\[ \frac{d}{dx} \left( x^2 \right) = 2x \]
\[ \int \frac{x^3}{x^4 + 1} \, dx \]

Let \( u = x^4 + 1 \)

\[ du = 4x^3 \, dx \]

\[ \frac{1}{4} \, du = x^3 \, dx \]

\[ \frac{1}{4} \int \frac{u^{-4}}{u^2 + 1} \, du \]

\[ = \frac{1}{4} \int \frac{u^{-3}}{u^2 + 1} \, du \]

\[ = -\frac{1}{2} \ln |u^2 + 1| + C \]

\[ = -\frac{1}{2} \ln |x^4 + 1| + C \]

\[ \int 12x^2 \sec x \, dx \]

Let \( u = x^2 \sec x \)

\[ du = 2x \sec x + x^2 \tan x \, dx \]

\[ 12 \int x^2 \sec x \, dx = u^3 + C = \frac{1}{3} u^3 + C \]
#11 \[ \int_{1}^{2} x^2 (x+1)(x^2+2x) \, dx \]

Set \( u = x^2 + 2x \)

\[ du = (2x+2) \, dx \]

\[ \frac{1}{2} du = (x+1) \, dx \]

\[ u = x^2 + 2x \]

\( x = 2 \quad u = 8 \)

\( x = 1 \quad u = 3 \)

\[ \frac{1}{2} \int_{3}^{8} u^3 \, du = \frac{1}{2} \int_{3}^{8} \, du \]

\[ = \frac{1}{2} \left[ \frac{u^4}{4} \right]_{3}^{8} \]

\[ = \frac{1}{8} 8^4 - \frac{1}{8} 3^4 \]

#12 \[ \int_{1}^{2} \sqrt{5x+6} \, dx \]

Set \( u = 5x+6 \)

\[ du = 5 \, dx \]

\[ \frac{1}{5} du = dx \]

\[ u = 5x+6 \]

\( x = 2 \quad u = 16 \)

\( x = 1 \quad u = 11 \)

\[ \frac{1}{5} \int_{11}^{16} \sqrt{u} \, du = \frac{1}{5} \int_{11}^{16} u^{\frac{1}{2}} \, du \]

\[ = \frac{2}{15} \left( 16^{\frac{3}{2}} - 11^{\frac{3}{2}} \right) \]

\[ = \frac{2}{15} \left( 16^{\frac{3}{2}} - 11^{\frac{3}{2}} \right) \]