2018 John O'Bryan Mathematical Competition
Questions for the Two-Person Speed Event

Calculators may not be used on the first four questions

1. Let $k = \frac{x}{y}$ if $3^{x+4y} = \left(\frac{1}{9}\right)^{y-2x}$. Let $w = ab$ if $(a+b)^2 = 25$ and $a^2 + b^2 = 13$. Find the reduced, simplified form of $(k + w)$.

2. Define the universal set $U = \{0,1,2,3,4,5,6,7,8,9\}$. Let A be the set of possible unit digits in the square of any integer and B be the set of possible unit digits in the cube of any integer. Find the set $\overline{A} \cap B$ (the complement of A and B in U). Express your answer using set notation.

3. Let 2 and 10 be the first and third terms respectively of a sequence. k represents the 7th term if the sequence is arithmetic. w represents the 7th term if the sequence is geometric. Find $(k + w)$.

4. Let $a = \left|7 - 3\sqrt{6}\right| - \left|2\sqrt{6} - 4\right|$. Let $b = \left\lfloor\sqrt{18} - 2\pi\right\rfloor$. Find the exact value of $(a + b)$. Express your answer as a simplified radical expression. (Note: $\lfloor x \rfloor$ represents the greatest integer, or floor, function of x.)

Calculators may be used on the remaining questions

5. If $p \leq q \leq r$ are the lengths of the three sides of a triangle whose area is 1, let k be the exact minimum value for q. The line $y = 3x + b$ is tangent to the circle $x^2 + y^2 = 10$. Let w be the larger of the two possible values for b. Find the exact value of $(kw)^2$.

6. $\triangle ABC$ is a right triangle with hypotenuse of length 17.5, one leg of length 10.5, and the other leg of length d. $\begin{vmatrix} 3 & t & 5 \\ 2 & 0 & 3 \\ 4 & 1 & 2 \end{vmatrix} = 65$. Find the sum $(d + t)$.

7. A triangle with sides of lengths 12, 35, and 37 is inscribed in a circle. Let A be the numerical area inside the circle but outside the triangle. Let T be the numerical length of the altitude to the longest side of the triangle. Find the numerical sum $(A + T)$. Express your answer as a decimal rounded to the nearest tenth.

8. Two points are chosen randomly from the circumference of a circle. P is the probability that the chord connecting these points is longer than the radius of the circle. A regular hexagon is inscribed in the circle. Q is the probability that a point chosen at random from inside the circle also lies inside the hexagon. Find the product (PQ). Express your answer as a decimal rounded to the nearest thousandth.

9. (Tiebreaker #1) The function $f(x)$ has a maximum value at coordinates $(2,12)$. Find the coordinates of the minimum of the function $-2f(x + 3)$. Express your answer as an ordered pair (x, y).

10. (Tiebreaker #2) One rectangular solid has sides of lengths 3, 4, and 12. What is the length of its main diagonal?

Name: _____ANSWERS_____   Team Code: _____

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Answers for the Two-Person Speed Event

Note: All answers must be written legibly and in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required. Each problem has the same point-value; however ties for individual awards will be broken based on problem difficulty.

1. _____8_____

2. _____$\{2,3,7,8\}$_____   **Must be in set notation; any order.**

3. _____276_____

4. _____$-6 + \sqrt{6}$ or $\sqrt{6} - 6$_____   **Must be one of these exact answers.**

5. _____200_____

6. _____22_____

7. _____876.6_____

8. _____0.551_____

T1. _____$(-1, -24)$_____   **Must be this ordered pair.**

T2. _____13_____

> **Calculators are not allowed to be used on the first four questions!**

> This competition consists of eight competitive rounds. Correct answers will receive the following scores:
>
> 1st: 7 points
> 2nd: 5 points
> All Others: 3 points
>
> There is a three minute time limit on each round. You may submit only one answer each round. To submit your answer, fold this sheet **lengthwise** and hold it high in the air so that a proctor may check your answer.