2016 John O'Bryan Mathematical Competition
Junior-Senior Individual Test

Directions: Please answer all questions on the answer sheet provided. All answers must be written legibly and in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required. Each problem has the same point-value.

1. Find the **exact** distance between the centers of the two circles whose respective equations are $x^2-8x+y^2+6y=-16$ and $(x+5)^2+(y-1)^2=81$.

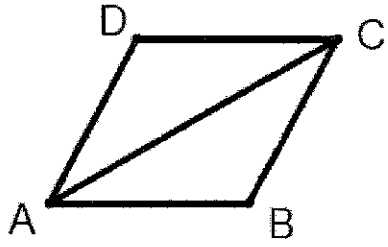
2. Let $i=\sqrt{-1}$ and let r represent a real number. If $r-5i+x=-7+2i$ is solved for x, then $x=-10+7i$. Find the value of r.

3. Given the arithmetic sequence $-10,-9,-8,...,10$, find the **sum** of all distinct members k of that arithmetic sequence for which $0.5^k<0.49$.

4. Let $A=\{8,4,2,j\}$. If j is equally likely to be any one of the first 100 positive integers, find the probability that A will consist of 4 distinct numbers that can be arranged in some order to form a geometric sequence. Express your answer as a **common fraction** reduced to lowest terms.

5. A bag has 1 green and 3 red marbles. You take out two marbles at the same time. What is the probability that they are the same color? Write your answer as a **reduced common fraction**.

6. In the diagram, $ABCD$ is a rhombus. If the degree measure of $\angle ADC$ is 7 times the degree measure of $\angle BAC$, find the degree measure of $\angle BCD$.



7. A trapezoid has sides with respective lengths 2, 41, 20, and 41. Find the length of an altitude of this trapezoid.

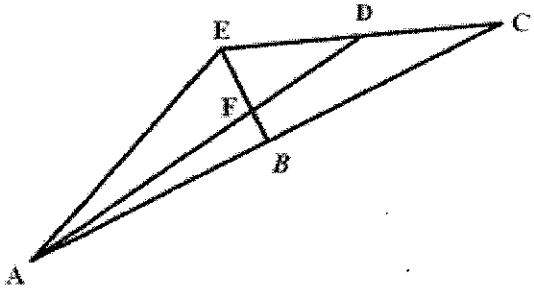
8. Find the value of $(\log_2 625)(\log_{25} 20)(\log_{20} 2)(\log_5 25)$.

9. In how many distinct points do the real graphs of $x^2+y^2=29103$ and $-x^2=29102y$ intersect?

10. Let $a>0$. Dealing with only real numbers, $\log_a(2x+6)=3$ and $\log_a(3x+54)=6$. Find the value of x. Express your answer as a **decimal**.

11. In $\triangle XYZ$ with right angle at Z, $\frac{XZ}{YX}=\frac{9}{41}$. Find $\sin(\angle YXZ)$. Express your answer as a **common fraction** reduced to lowest terms.

12. In the given diagram, $\overline{AB}\cong\overline{BC}$, $\overline{ED}\cong\overline{DC}$, $EF=FB+3$ and $AF=3(FD)-5$. How many units longer is $\overline{AF}$ than $\overline{FB}$?



13. Let n represent a positive integer such that $0<n<92$. If $n!$ is an integral multiple of 11, find the **sum** of all possible distinct values of n.

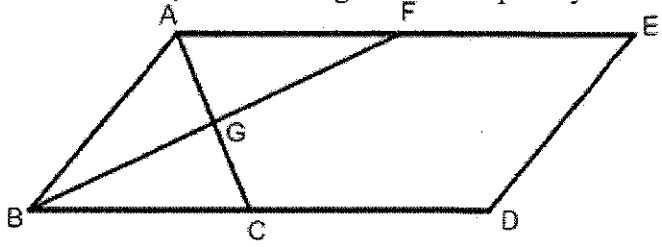
14. In a finite geometric sequence, the last term is 1458, the common ratio is -3, and the sum of the terms is 1094. Find the second term of this geometric sequence.

15. In a bridge game with a standard 52 card deck (four suits with 13 ranks each), each of 4 persons is dealt 13 cards at random. If Joe is one of these four persons, find the probability that Joe was dealt exactly 4 spades, exactly 4 hearts, exactly 2 diamonds, and exactly 3 clubs. Express your answer as a decimal rounded to 5 decimal places.

16. In $\triangle DEF$, $\sin(\angle EDF)+\cos(\angle EDF)=\frac{167}{145}$. If $\tan(\angle EDF)<1$, find $\tan(\angle EDF)$. Express your answer as a common fraction reduced to lowest terms.

17. If $\frac{(x+n)!}{x!}-x=1$ for all possible values of x, where x, n represent positive integers, find the value of n.

18. In the diagram, $ABDE$ is a parallelogram with F on $\overline{AE}$ and C on $\overline{BD}$. $\overline{AC}$ and $\overline{BF}$ intersect at G. If $AG=10$, $GF=12$, and $BG=2.88$, find the length GC. Express your answer as an **exact decimal**.



19. Stacie has 7 t-shirts, identical in every way except for color. Three of these shirts are the same color, while the other shirts are all different colors. In how many ways can she arrange these 7 t-shirts in a single row on a table?

20. Consider the sequence $\{x_n\}$ defined as $x_{n+1}=\frac{5x_n}{8}+\frac{126}{8x_n}$ for integral n such that $n\geq 1$ and for which $x_1=10$. As n increases without bound, the limiting value of x_n is k. Find the **exact** value of k.

Name: _____ Team Code: _____

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1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____
11. _____
12. _____
13. _____
14. _____
15. _____
16. _____
17. _____
18. _____
19. _____
20. _____

Name: _____ANSWERS_____ Team Code: _____

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Note: All answers must be written legibly and in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required. Each problem has the same point-value (1 point).

1. $\sqrt{97}$ Must be this exact answer
2. **3**
3. **54**
4. $\frac{1}{50}$ Must be this fraction.
5. $\frac{1}{2}$ Must be this fraction.
6. **40** Degrees Optional
7. **40**
8. **4**
9. **2**
10. **0.75** Must be this decimal
11. $\frac{40}{41}$ Must be this fraction
12. **7**
13. **4131**
14. **-6**
15. **0.01796** Must be this decimal
16. $\frac{24}{143}$ Must be this fraction
17. **1**
18. **2.4** Must be this decimal
19. **840**
20. $\sqrt{42}$ Must be this exact answer