MA-M-4.2.4

Problem-Solving Strategy
Look for a Pattern

Total Rectangles

1. What is the total number of rectangles, of any size, in the figure below?

2. What is the total number of rectangles, of any size, if a figure is made up of 6 small rectangles?

3. What would be the total number of rectangles, of any size, if a figure in made up of n (any number of) small rectangles?

Scoring Guide

Total Rectangles

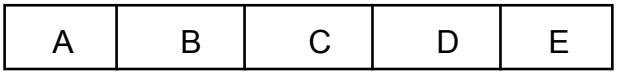
4	Completes all components. Communicates clearly. In-depth understanding. Thinks on a higher level.
3	Completes most components. Communicates clearly. Understands major concepts but overlooks some ideas and/or details.
2	Completes some components. Has gaps in understanding.
1	Minimal understanding. Addresses only a small portion of the task.
0	Response is totally incorrect or irrelevant.
Blank	No response.

1. Five small rectangles in a row gives you a total of **15** rectangles.

2. Six small rectangles in a row gives you a total of **21** rectangles.

3. $\frac{n(n+1)}{2}$ OR any variation of this rule

Solution 1



First, find the number of small rectangles in the figure.

A
B
C
D
E

Second, find the number of rectangles that can be made from two small rectangles.

AB
BC
CD
DE

Third, find the number of rectangles that can be made from three small rectangles.

ABC
BCD
CDE

Forth, find the number of rectangles that can be made from four small rectangles.

ABCD
BCDE

Fifth, find the number of rectangles that can be made from five small rectangles.

ABCDE

Finally, add the results from all of the steps above.

5 + 4 + 3 + 2 + 1 = 15

The same approach can be followed if you have six small rectangle in a row.

A B C D E F

Small rect.	Two small rect.	Three small rect.	Four small rect.	Five small rect.	Six small rect.
A	**AB**	**ABC**	**ABCD**	**ABCDE**	**ABCDEF**
B	**BC**	**BCD**	**BCDE**	**BCDEF**	
C	**CD**	**CDE**	**CDEF**		
D	**DE**	**DEF**			
E	**EF**				
F					

Again, add the results to get the total number of rectangles.

6 + 5 + 4 + 3 + 2 + 1 = 21

I looked at the data from the previous two questions

5 small rectangles in a row

Rectangles made with	
1 small rectangle	5
2 small rectangles	4
3 small rectangles	3
4 small rectangles	2
5 small rectangles	1

6 small rectangles in a row

Rectangles made with	
1 small rectangle	6
2 small rectangles	5
3 small rectangles	4
4 small rectangles	3
5 small rectangles	2
6 small rectangles	1

I noticed that the left side starts at one and counts up by whole numbers to the number of small rectangles that you have in a row and the right side starts with the number of small rectangles that you have in a row and counts down by whole numbers to 1.

In order to find the total number of rectangles in any figure made up of small rectangles all you have to do is find the sum of the whole numbers starting with the number of small rectangles you are given in the row down to one. (The more rectangles that you are given in the row – the longer it will take you to find the sum.)

Solution 2

I looked at simpler problems and organized my findings into a table.

No. of rectangles in a row	Total no. of rectangles	No. of each size rectangle
1	1	1
2	3	2 + 1
3	6	3 + 2 + 1
4	10	4 + 3 + 2 + 1
5	15	5 +4 + 3 + 2 + 1

I noticed that between the 1st and 2nd rectangle you add 2, between the 2nd and 3rd rectangle you add 3, between the 3rd and 4th rectangle you add 4, and between the 4th and 5th rectangle you add 5.

In order to calculate the total number of rectangles in a figure made up of six small rectangles I can add 6 to 15 (the answer to the total no. of rectangles for 5 small rectangles in a row). The result would be 21..

6	21	6 + 5 + 4 + 3 + 2 + 1

In order to come up with some type of rule in order to get the answer quicker I started looking for relationships between the numbers in the first two columns of my table.

1 x ? = 1 ? = 1

2 x ? = 3 ? = 1.5

3 x ? = 6 ? = 2

4 x ? = 10 ? = 2.5

5 x ? = 15 ? = 3

6 x ? = 21 ? = 3.5

As I filled in each ? I started to ask myself how is the ? related to the beginning number?
I discovered that the ? was really related to the next whole number following the beginning number. In fact, it was half of the next whole number. This lead me to the following rule: To find the total number of rectangles in a figure made up on any number of small rectangles, you multiply the number of small rectangles by half of the next whole number. **n x 0.5(n+1) or [n(n+1) ÷ 2]**

Solution 3

I looked at simpler problems and organized my findings into a table.

No. of rectangles in a row	Total no. of rectangles	No. of each size rectangle
1	1	1
2	3	2 + 1
3	6	3 + 2 + 1
4	10	4 + 3 + 2 + 1
5	15	5 +4 + 3 + 2 + 1

I noticed that in order to find the total no. of rectangles you can find the sum of all the whole numbers from the no. of rectangles you have in a row down to the whole number one.

In order to calculate the total number of rectangles in a figure made up of six small rectangles I can add the whole numbers from six down to one and get the result of 21.

6	21	6 + 5 + 4 + 3 + 2 + 1

In order to come up with some type of rule in order to get the answer quicker I started looking for relationships between the numbers in the first two columns of my table.

1 x ? = 1 ? = 1

2 x ? = 3 ? = 1.5

3 x ? = 6 ? = 2

4 x ? = 10 ? = 2.5

5 x ? = 15 ? = 3

6 x ? = 21 ? = 3.5

As I filled in each ? I started to ask myself how is the ? related to the beginning number?
I discovered that the ? was related to the number in the following way. You can take half of the number of rectangles in a row and add 0.5 to it and you end up with the ?. **Let n = the number of small rectangles in the row. To find the total number of rectangle you can use the following formula: n x (0.5n + 0.5)**