Pentominoes Symmetry

8th Grade Open Response Question

Academic Expectations:

1.5 Students use mathematical ideas and procedures to communicate, reason, and solve problems.

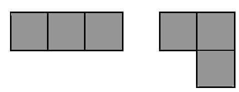
2.9 Students understand space and dimensionality concepts and use them appropriately and accurately.

Kentucky Core Content:

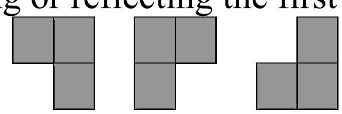
Students will describe properties of, define, give examples of, and/or apply to both real-world and mathematical situations:
MA-M-2.1.4 *congruence, symmetry, and similarity.*

THE PROBLEM

A pentomino is like a domino, but has 5 squares instead of 2. The squares must be arranged to share a side. Examples of triominoes (made with 3 squares) are shown in the shaded areas here:



Note that any *rotation* or *reflection* of the triomino is still the same shape. For example, the following triominoes are all considered the same shape, because simply rotating or reflecting the first triomino produces the others.



These are all the same. Do not count a rotation or reflection as a new shape!

a) Using graph paper, color all the pentominoes (5 squares which share at least one side) possible. Label each shape with a letter. Be careful *not* to count rotated or reflected pentominoes as different shapes.

b) List all pentominoes containing at least one line of symmetry.

c) Indicate the total line(s) of symmetry for each of the symmetric pentominoes. You may draw the lines or give the number of lines of symmetry.

d) Consider the result of rotating each pentomino. Which pentominoes give an identical image (compared to the original image) when rotating less than 360 degrees? Give a generalization for the rotational result of any shape, based on your conclusion about the pentominoes.

Written by Alice Gabbard

Name___________________________

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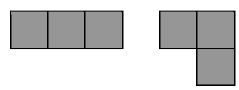

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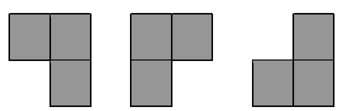

These are all the same. Do not count a rotation or reflection as a new shape!

a) Using graph paper, color all the pentominoes (5 squares which share at least one side) possible. Be careful *not* to count rotated or reflected pentominoes as different shapes.
b) List all pentominoes containing at least one line of symmetry.
c) Indicate the total line(s) of symmetry for each of the symmetric pentominoes. You may draw the lines or give the number of lines of symmetry.
d) Consider the result of rotating each pentomino. Which pentominoes give an identical image (compared to the original image) when rotating less than 360 degrees? Give a generalization for the rotational result of any shape, based on your conclusion about the pentominoes.

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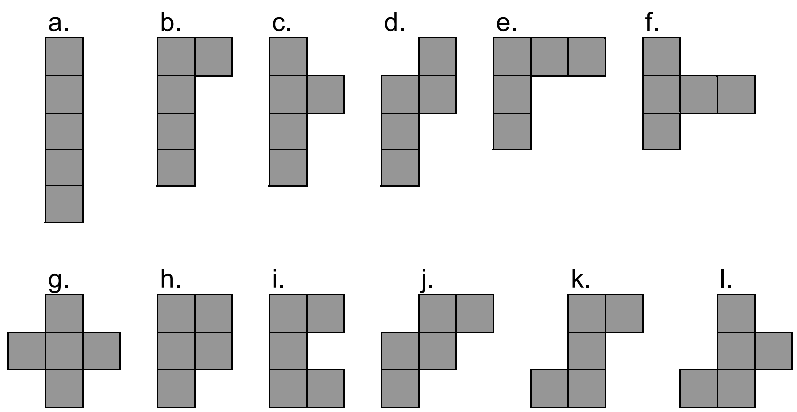
SCORING GUIDE

4	Student earns 4 points.
3	Student earns 3 points.
2	Student earns 2 points.
1	Student earns 1 point.
0	Student earns 0 points.



Part a) Student correctly draws 5-6 pentominoes—0.25 pts.
Student correctly draws 7-8 pentominoes—0.5 pts.
Student correctly draws 9-10 pentominoes—0.75 pts.
Student correctly draws 11-12 pentominoes—1 pt.

Part b) Student correctly lists 3-4 pentominoes with symmetry—0.5 pts.
Student correctly lists 5-6 pentominoes with symmetry—1 pt.

Part c) Student correctly finds the total lines of symmetry for 3-4 pentominoes—0.5 pts.
Student correctly finds the total lines of symmetry for 5-6 pentominoes—1 pt.

Part d) Student indicates that the pentominoes with more than 1 line of symmetry can be rotated less than 360 degrees to produce an identical image of the original—0.5 pt.
Student gives a logical generalization for all shapes, based on the conclusions stated earlier in part d.—0.5 pt.

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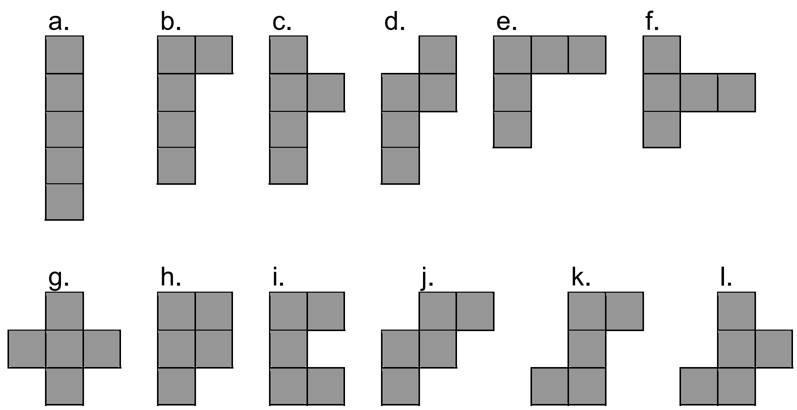
Distinguished Student Response

Part a.



Parts b. & c.

Pentominoes with Symmetry	
Pentomino	Total Lines of Symmetry
a.	2
e.	1
f.	1
g.	4
i.	1
j.	1

Part d.

When I started rotating each pentomino in my mind, I found that most of them only look the same when they end up where they started—that is, they have to be rotated 360 degrees to produce an identical image. The two pentominoes containing more than 1 line of symmetry, a. and g., are the only ones that produce an identical image by rotating less than 360 degrees. Pentomino a. has 2 lines of symmetry and produces an identical image when rotated 180 degrees. Pentomino g. has 4 lines of symmetry and produces an identical image when rotated every 90 degrees. It is interesting that the rotation angle is the same as 360/# lines of symmetry. My generalization is that any shape with more than 1 line of symmetry can be rotated less than 360 degrees to make an identical image of the original. Further, the degrees of rotation are equal to 360/# lines of symmetry.

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