

## Grade 8 – Mathematics

The **academic expectations** addressed by “Octagonal Trains” are

- 2.7 Students understand number concepts and use numbers appropriately and accurately.
- 2.8 Students understand various mathematical procedures and use them appropriately and accurately.
- 2.11 Students understand mathematical change concepts and use them appropriately and accurately.

The **core content** assessed by this item includes

### Algebraic Ideas Concepts

- MA-M-4.1.1 Students should be able to describe properties of, define, and give examples of variables, equations, inequalities, and algebraic expressions.

### Algebraic Ideas Skills

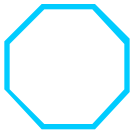
- MA-M-4.2.2 Students should be able to solve simple equations and inequalities.
- MA-M-4.2.3 Students should be able to model equations and inequalities concretely, pictorially, and abstractly.
- MA-M-4.2.6 Students should be able to represent and use functions through tables, graphs, verbal rules, and equations.

# Open Response Question

## Octagonal Trains

The drawing below shows “trains” of regular octagons made from toothpicks

One Octagon  
Train



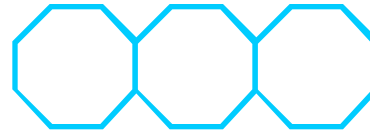
1<sup>st</sup> Train

Two Octagon  
Train



2<sup>nd</sup> Train

Three Octagon  
Train



3<sup>rd</sup> Train

— Equals one toothpick

- A. How many toothpicks will it take to make 8 regular octagon trains? Explain how you arrived at your answer.
- B. Write an expression that describes the relationship between the number of regular octagons in the  $n^{\text{th}}$  train and the number of toothpicks. Explain how you found your expression.
- C. How many toothpicks would you need to build the 120<sup>th</sup> train? Explain how you arrived at your answer.
- D. How would the expression change in part B if the trains were in the shape of a square? A regular hexagon? Explain how you arrived at your answer.

## Scoring Guide Grade 8 Mathematics

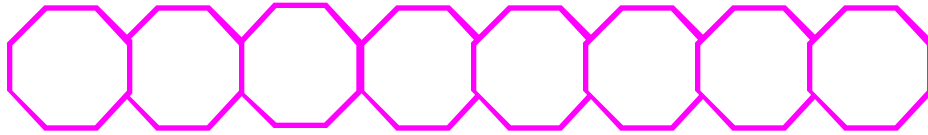
Score	Description
4	Student answers all of parts a, b, c, and d correctly and explains how he/she arrived at his/her answer for all four parts.
3	Student answers all of parts a, b, c, and d correctly, but does not explain how he/she arrived at his /her answer for one or more parts. OR Student answers all of three of the four parts correctly and explains how he/she arrived at his/her answer for three or more parts.
2	Student answers all of three of the four parts correctly, but does not explain how he/she arrived at his/her answer for three or more parts. OR Student answers all of two of the four parts correctly and explains how he/she arrived at his/her answer for two or more parts.
1	Student answers all of two of the four parts correctly and explains how he/she arrived, but does not explain how he/she arrived at his/her answer for two or more parts. OR Student answers all of one of the four parts correctly and explains how he/she arrived at his/her answer for one or more parts. OR Student answers all of one of the four parts correctly and does not explain how he/she arrived at any of the answers.
0	Response is totally incorrect or irrelevant.
Blank	No response.

Correct Answers:

- a. 57 toothpicks
- b.  $7n + 1$  or  $7(n-1) + 8$  or another equivalent expression ( $n$  = the train number or the number of octagons in the train)
- c. 841 toothpicks
- d. For a square the constant of  $n$  would change to three so the expression would be  $3n + 1$  (or equivalent expression where  $n$  = the train number or the number of squares in the train)  
For a hexagon the constant of  $n$  would change to five, so the expression would be  $5n + 1$  (or equivalent expression where  $n$  = the train number or the number of hexagons in the train)

## Sample 4-Point (Distinguished level) Response of Student Work

A.



$$8 + 7 + 7 + 7 + 7 + 7 + 7 + 7 = 57$$

toothpicks

— Equals one toothpick

There are a total of 57 toothpicks needed for a train with 8 octagons. The way I arrived at my answer is I drew the first octagon and then I added the second octagon. When I added the second octagon, I saw that I would only need 7 toothpicks for the second octagon because the left side was already there from first octagon. I then drew a third octagon. Again I saw that I would only need 7 toothpicks for the third octagon because the left side was already there from the second octagon. I continued this process until I had eight octagons. Then I added up the total number of toothpicks and the answer is 57 toothpicks.

B.

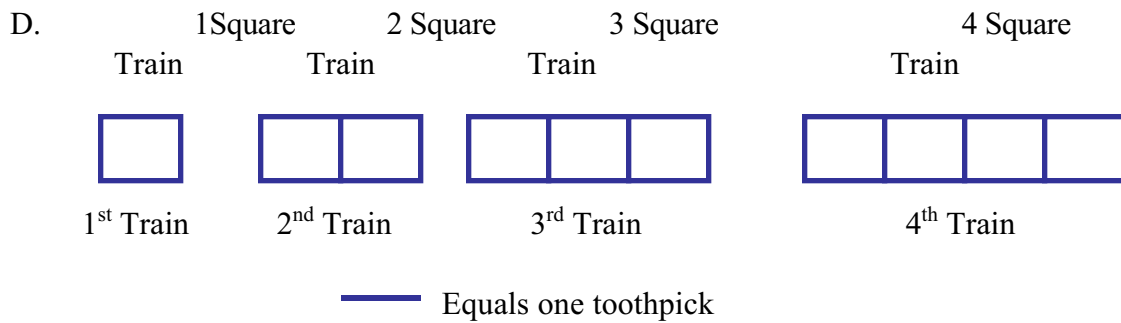
Train # (# of Octagons)	# of Toothpicks needed
1	8
2	$8 + 7 = 15$
3	$8 + 7 + 7 = 22$
4	$8 + 7 + 7 + 7 = 29$
5	$8 + 7 + 7 + 7 + 7 = 36$
.	.
.	.
n	$7n + 1$

The number of toothpicks need for the  $n^{\text{th}}$  train would be  $7n + 1$ , where  $n$  is the train number (or the number of octagons together). I found my answer by making a table. I compared the train number (number of octagons) to the number of toothpicks needed to make that train. I saw the pattern that each time 7 more was being added. I then looked at the final answer for the number of toothpicks and compared that to the train number. I noticed that if you took 7 times the train number and add one, you had the number of

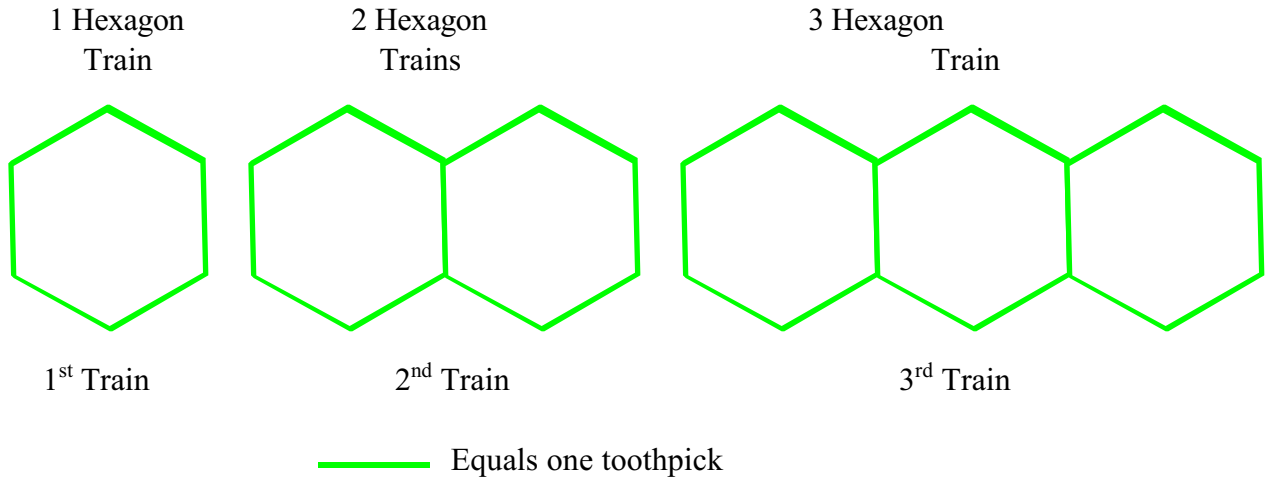
toothpicks needed. This made sense because every octagon except for the first one needs 7 toothpicks. The first one needs one extra toothpick... that is why you have to add one more.

C.  $7(120) + 1 = 841$  toothpicks

The number of toothpicks needed for the 120<sup>th</sup> train is 841. I found this answer by plugging 120 into my expression that I found in part c.



Train # (# of Squares)	# of Toothpicks needed
1	4
2	$4 + 3 = 7$
3	$4 + 3 + 3 = 10$
4	$4 + 3 + 3 + 3 = 13$
5	$4 + 3 + 3 + 3 + 3 = 16$
.	.
.	.
n	$3n + 1$



Train # (# of Hexagons)	# of Toothpicks needed
1	6
2	$6 + 5 = 11$
3	$6 + 5 + 5 = 16$
4	$6 + 5 + 5 + 5 = 21$
5	$6 + 5 + 5 + 5 + 5 = 26$
.	.
.	.
n	$5n + 1$

I found that the constant of n is the part that would change when the shapes are changed to squares or hexagons. For a square, the expression for the n<sup>th</sup> train would be  $3n + 1$ . I found this by first drawing the squares and then I made a table. I saw that the squares had the same pattern as the octagons. From one train to the next, you had to add three toothpicks. This is because the left side was already there, so you only had to add three more sides. Therefore the expression is  $3n + 1$ , where n is the train # (or the number of squares together). I did the same thing to check the hexagons. I found this expression by drawing the hexagon trains and by making a table of the data. From one train to the next you had to add 5 toothpicks because the left side was already there. Therefore the expression is  $5n + 1$ , where n is the train # (or the number of hexagons together). I also discovered this will work for any regular polygons that you can put together and share one side. The key here is they can only share one side. If the shapes share no sides or more than one side then this expression will not work.