
LESSON PLAN I

THE MILLION DOLLAR JOB

DEMOGRAPHIC INFORMATION:

Name: Heather Dunn
Date: September 11, 2001
Lesson Length: 1 day with assessment
School: Woodland Middle School
Grade Level: 8th Grade
Number of Students: 23-26
Subject: Pre-Algebra, Algebra I
Topic: Exponents, Doubling
Number of IEPs: 0

OBJECTIVES:

Students will work in groups to create a theory and prove it by problem solving while understanding the concept of doubling.

- Students will examine the problem to determine if one cent doubled each day for a month is more than a million dollars.
- Students will explore the problem in numerous ways such as creating a table, finding a pattern, or formula to solve the problem.
- Students will then describe what they found and determine if their original hypothesis was correct.

PROGRAM OF STUDIES DEMONSTRATOR:

Students will recognize, create, and continue patterns (generalize the pattern by giving the rule for the n th term and defend the generalization).

CORE CONTENT FOR ASSESSMENT DEMONSTRATORS:

- Students will describe properties of, define, give examples of, and/or apply to both real-world and mathematical situations with positive whole number exponents.
- Students will perform the following mathematical operations and/or procedures accurately and efficiently, and explain how they work in real-world and mathematical situations by identifying and using number theory concepts (multiples) to solve problems.

CONNECTIONS:

My objectives relate to the following Kentucky Learner Goals:

Learner Goal 1.5 – 1.9

Students use mathematical ideas and procedures to communicate, reason, and solve problems.

- Students will solve the problem in various ways using graphs, guess and check, or looking for patterns to create an equation to solve for n .

Learner Goal 2.7

Students understand number concepts and use numbers appropriately and accurately.

- Students will demonstrate how the equation they created works to solve the problem. From this they will explain why one penny doubled every day for a month is greater than a million dollars.

NCTM PRINCIPLES AND STANDARDS FOR SCHOOL MATHEMATICS:

My objectives relate to the following NCTM Principles and Standards for School Mathematics because this is an 8th grade class that is just starting to explore Algebra 1. These objectives are a result of NCTM standards for grades 6-8.

1) Number and Operations:

Understand numbers, ways of representing numbers, relationships among numbers, and number systems:

- Develop an understanding of large numbers and recognize and appropriately use exponential, scientific, and calculator notation.

Using the calculator and/or a table, students should find a relationship between the numbers to create an equation using exponents.

2) Algebra Standard:

Understand patterns, relations, and functions:

- Represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules.

Using the patterns they found, they should create an equation using symbolic symbols in order to solve for n .

3) Problem Solving Standard:

Apply and adapt a variety of appropriate strategies to solve problems.

Students can use various ways to solve the problem with a table, linear math, or check and guess. These various ways will be examined in class to show that there is more than one way to solve a problem.

4) Communication Standard:

Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.

Students will demonstrate their work on the board and discuss the problem in a group to clearly explain the various ways of solving the problem.

CONTEXT:

The goals for teaching Algebra are based on the Kentucky Learner Goals and the Core Content for Assessment under the Algebra Strand. My objectives relate to these goals and have been reviewed above.

Students need to understand that math includes many ways of solving problems. They should ask questions and try different ways to come to a conclusion. They should also work with their peers to come up with more ways to solve a particular problem. By creating tables or using a calculator they can come up with patterns in order to solve problems. Student should realize it is not always necessary to figure out a problem by computing the doubling of a penny from the first day to the thirtieth day. At this point we can discuss how Algebra can help to solve various problem using just one equation.

This lesson can then lead into the additional goals of Algebra such as creating algebraic expressions and using them to solve problems. They will also learn how to further use their calculator to develop these problems.

To prepare students for this lesson we will go over how to compute exponents. We will also examine how to use the calculator to solve exponents. The students already have a background in using exponents. They know how to perform various functions and problem solving using exponents. This unit was taught in the 7th grade and reviewed 2 weeks ago.

MATERIALS/ TECHNOLOGY:

Overhead transparencies, calculators, paper, pencil, handout with the problem on it.

PROCEDURES:

The first thing I will do is pass out the handout. The next step will include reading the problem with the class off the overhead projector. Please refer to the handout that is

attached. The transparency will be a copy of the handout. After reading the problem I will go over the directions step by step.

1. Write down which option you think produces the most money.

It will be explained to the students that it is ok if they select the wrong option. Don't change your answer later, just keep it.

2. Prove your theory. You can do this in many different ways to get the correct answer. Write down everything you use to find out if option 1 or 2 gives you the most money.

After you state a hypothesis (what they did first) you must prove it. I'm not going to give them examples here, because I want to see how they do first. Some students should be able to come up with a way to solve it on their own. If I see students having trouble, I will give them advice on how to continue. I will also explain that I want them to write everything down. Any patterns they see whether they think it is significant or not should be written down. They may find it useful later.

3. As you are trying to figure out which way produces the most money, look for a pattern. We are trying to find a simple equation to use in order to solve how much money you will have at the end of thirty days.

At this time I will discuss that as they are looking to find which option produces the most money. See if you can produce an equation to make it simple. Look for patterns in your multiplication. I will give an example of what a pattern looks like, such as 2,4,6,8 increases by 2. I will also write down what an equation looks like, such as $5X - 10$.

After I complete the directions, I will ask for any questions. Then I will have the students begin to solve it. After about 5 minutes of working, I will have the students get into groups of 2 or 3. As they are working, I will walk around the room to help or provide additional questions to think about. If a group solves the problem and gets the equation, they will then ponder what the amount would be after 60 days and what the equation would be if I started with a nickel.

When every group has reached an answer, I will get the class together to discuss what different problem solving skills they used. I will have various groups demonstrate on the board and discuss why it worked. This will show the class that more than one way is acceptable to solve the problem. I will then find out if anyone found the equation and how he or she found it. This will get them ready for Algebra.

I will end the class by showing them a transparency of a sales tool. The transparency is a copy of a brochure that financial planners use to show people how fast they can make money by investing in funds. This will give them that final tie into how math is used outside the classroom. A copy of this is attached. It is the exact same problem that the students did in class.

STUDENT ASSESSMENT:

See the attachments for the problem, answer sheet, and rubric.

I will assess student progress and understanding by walking around the class to watch them work. Students will be asked questions as they work to find out the level of understanding. When the lesson is complete, students will hand in their work. I will then evaluate the written work based on the rubric.

REFERENCES:

Lanius, Cynthia (1997-1999). *Cynthia Lanius' Lesson: A Mathematics Fantasy*.

Retrieved September 2, 2001, from Rice University, Math Department web site:

<http://math.rice.edu/~lanius/pro/rice.html>

Dr. Math (1994-2001). *Math Forum: Ask Dr. Math, Doubling Pennies*. Retrieved

September 2, 2001, from Drexel University, Math Forum website:

<http://forum.swarthmore.edu/dr.math/faq/faq.doubling.pennies.html>

THE MILLION DOLLAR JOB

NAME: _____

How would you like to have a million dollars?

You are sitting in math class, when in walks Ms. Smith. Ms. Smith is a local talent scout looking for someone to star in a new film. If you are selected you will be working for the next 30 days. You have a choice on how you will get paid:

1. One cent on the first day, two cents on the second day, and double your salary every day thereafter for the a total of thirty days, or
2. Exactly 1 million dollars, that's \$1,000,000.

Which option would you choose?

MISSION:

Your mission is to find out which option will give you the most money.

1. Write down which option you think produces the most money.
2. Prove your theory. You can do this in many different ways to get the correct answer. Write down everything you use to find out if option 1 or 2 gives you the most money.
3. As you are trying to figure out which way produces the most money, look for a pattern. We are trying to find a simple equation we can use to solve the problem (hint: it will include exponents).

Take your time and remember to write everything down.

Answer Sheet:

1. The problem can be solved using a chart:

DAY	PAY FOR THE DAY
1	1
2	2
3	4
4	8
5	16
6	32

AND SO ON TILL YOU GET TO 30 DAYS AND 536,870,912 pennies

The pennies will then have to be added up to get the total amount of pennies. The total amount is 1,073,741,824. The amount will then have to be divided by 100 in order to get the dollar amount. Since there are 100 pennies in a dollar. The dollar amount is \$10,737,418.23. Or the students could create another column of how much money they have as they go along.

2. It can be solved as a linear equation:

Day 1: 1

Day 2: $1+2 = 3$

Day 3: $1+2+4 = 7$

Day 4: $1+2+4+8 = 15$

Day 5: $1+2+4+8+16 = 31$

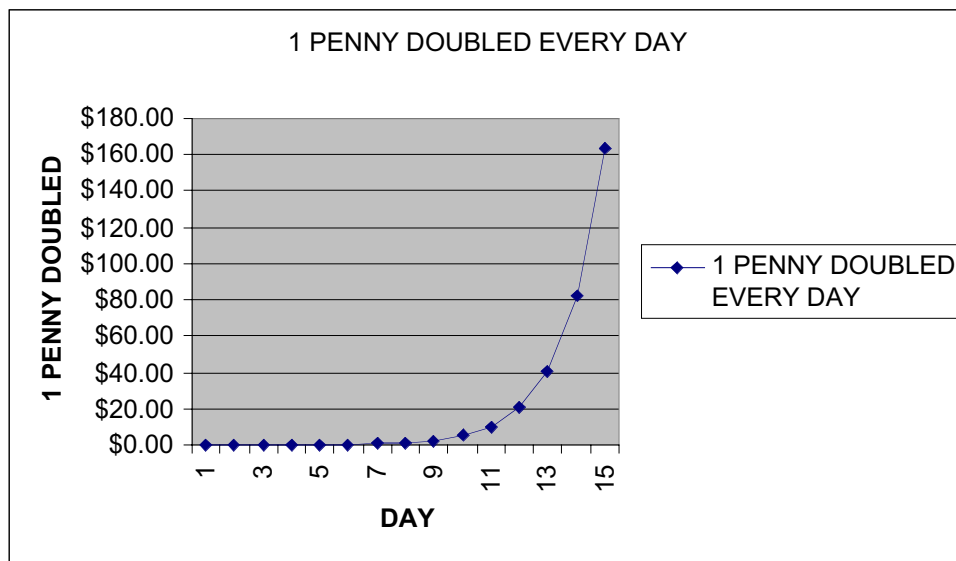
They can keep this up for 30 days. Or I would hope they would see a pattern at this point. The equation involves 2 being squared. This method would be a really good way to see the equation $2^a - 1 =$ the number of pennies. This will also produce an amount of pennies that will need to be divided by 100.

3. You can also solve it by creating a chart that will look for patterns.

Day	Number of Pennies	Total Pennies	Pattern
1	1	1	
2	$1 \times 2 = 2$	3	increase 2
3	$2 \times 2 = 4$	7	increase 4
4	$4 \times 2 = 8$	15	increase 8
5	$8 \times 2 = 16$	31	increase 16

Ok I see a pattern it is squaring, so I have a 2^a . To get the total number of pennies I need to subtract 1. Your formula would now look like, $2^a - 1 =$ number of pennies. To get the dollar amount divide by 100.

4. You could also create a graph and plot the amount of pennies against the days. You would see patterns on that.



This is for 15 days. The students could make a graph like this using a ruler, paper and pencil.

Scoring Rubric to Encourage Dept and Creativity:

Assessment Criteria	1	2	3	4
Dept of Understanding	Little or no understanding of the problem	Partial understanding; minor mathematical errors, found which one produced the most money	Good understanding; mathematically correct, found many patterns, and more than one way to do problem	In-dept understanding; well-developed ideas, various ways to do problem, found the equation
Fluency	One incomplete or unworkable approach	At least one appropriate way to solve the problem	At least two appropriate approaches and various patterns shown	Several appropriate approaches that lead to a equation
Generalizations	No generalizations made, or they are incorrect and reasoning is unclear	At least one correct generalization made; may not be well-supported with clear reasoning	At least one well-made, supported generalization, or more than one correct but unsupported generalization	Several well-supported generalizations; clear reasoning
Flexibility	No approaches used	All approaches use the same method (all graphs, all charts, all algebraic equations)	At least two methods of solution (graph, chart, equation)	Several methods of solution (graph, chart, equation)

IMPACT-REFLECTION/ANALYSIS OF TEACHING AND LEARNING

The objectives were met in the following ways:

- All of the students understood how to double a penny and found it was more than a million dollars. During core 1, chips had to be used by one student in order to understanding the doubling visually. Once the chips were manipulated he understood what was happening.
- As a group they found more than one way to solve the problem. But on the individual papers they only showed one way.
- They explained how they did the problem and why option 1 was a better choice during the whole group conversation.

PROGRAM OF STUDIES DEMONSTRATOR:

The students recognized that they were doubling the pennies. Some students also found the use of an exponent to create an equation. As I walked around the room they all saw the pattern of multiplying by 2. Only a select few came up with the complete equation. Samples of student work have been included. Referrer to this work to see how this was shown.

CORE CONTENT FOR ASSESSMENT DEMONSTRATORS:

The students used multiplication to find out that doubling a penny was more than a million dollars. Once this was done, some students saw the use of an exponent to get the same answer. I then showed the students an example of how this would work in the real world. The example is attached. It concerns investing a penny and having it doubled every day.

KENTUCKY LEANER GOALS:

The students as a group used various methods, but they did not do this individually. Some of the students found patterns and were able to form an equation or start an equation based on what they saw. None of the students that created the equation went further to show how it worked. So as a group, I explained how to use the equation to find the amount of money for sixty days.

NCTM PRINCIPLES AND STANDARDS:

All of the students worked well with the large numbers. I only had a few mathematical mistakes. I also did not have to show anyone how to use the calculator. The students that used exponents knew how to use the calculator. Even students without a scientific calculator knew you had to multiple out ($2 \times 2 \times 2 \times 2 \dots$) to get the answer for an exponent.

Only a few students created an equation. As a group they all knew what to do with the equation once I showed it to them. They could see that the n th term could be replaced with any number to get an answer.