

Lesson Plan
Tracy McManama
Sheffield - EDU 599

Demographic Information:

NAME: Mrs. McManama

DATE: 5 – 29 – 01

SCHOOL: Woodland Middle School

AGE/GRADE LEVEL: 11 & 12 years old
6th grade (Accelerated Class)

SUBJECT: Mathematics

TOPIC: Area of Circles

LESSON LENGTH: 55 minutes

NUMBER OF STUDENTS: 31

NUMBER OF IEP STUDENTS:

0

Actions:

GOALS: Students are able to use basic mathematics skills for purposes and situations they will encounter throughout their lives.

Students are able to use formulas to find measurements of two-dimensional shapes such as circles, quadrilaterals, and triangles.

OBJECTIVE: Students will be able to use mathematical strategies to compare areas of circles when given the diameter.

Program of Studies Demonstrator: Students will be able to use formulas to find area and perimeter of triangles, and quadrilaterals, area and circumference of circles, and surface area and volume of rectangular prism.

Core Content for Assessment Demonstrator: Students will be able to find circle measurements (radius, diameter, circumference, area) and relationships among them.

Connections to Kentucky Learner Goals:

- 1.5 -1.9 Students use mathematical ideas and procedures to communicate, reason, and solve problems.
- 2.8 Students understand various mathematical procedures and use them appropriately and accurately.
- 2.10 Students understand measurement concepts and use measurements appropriately and accurately.

Connections to NCTM Principals & Standards for School Mathematics:

Measurement Standard: Students will be able to develop and use formulas to determine the circumference of circles and the area of triangles, parallelograms, trapezoids, and circles and develop strategies to find the area of more complex shapes.

Geometry Standard: Students will be able to understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects.

Problem Solving Standard: Students will be able to apply and adapt a variety of appropriate strategies to solve problems. Students will also be able to monitor and reflect on the process of mathematical problem solving.

CONTEXT: Students will continue their study on measurement. Students have previously learned how to identify the parts of a circle and how to find the circumference and area of circles. In this lesson, students will use basic mathematics skills for purposes and situations they will encounter throughout their lives. The students will be presented with a problem that happens in everyday life. They will be looking at the prices of pizza and how diameter & radius sizes affect the area of the pizza. Students will use the formula for the area of a circle to compare pizzas.

MATERIALS/TECHNOLOGY: Fake ten-dollar bills, two 7-inch and one 14-inch pizza cut-outs, chalk, calculators, open-ended problem handout, and overhead & class copies of the scoring rubric.

PROCEDURES:

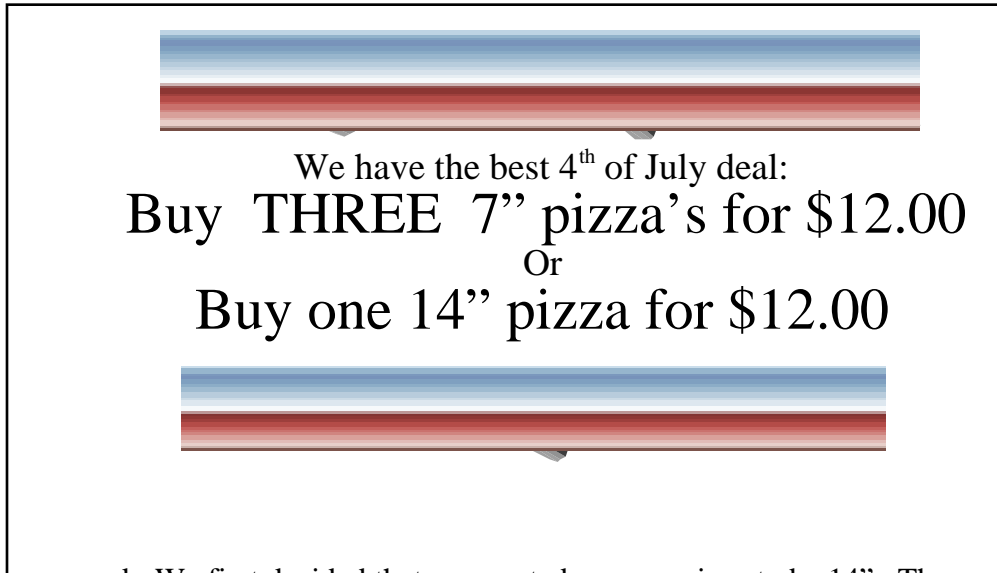
1. Give each student a ten-dollar bill. Tell them they have a choice of buying two 9" pizzas with their ten dollars or they can purchase one 14" pizza for the same price. Allow students to choose what deal they want.
2. Ask students, "What do we have to know in order to find out which deal is better?" Allow students to respond.
3. Review the parts of the circle by allowing students to come up to the board and draw the parts on the circle. Also review exponents and how to find the circumference and area of circles.
4. Discuss real life situations when you need to know the circumference and area of circles.
5. Discuss the question from the beginning of the lesson. "Is it better to buy two 9" pizzas for ten dollars or one 14" pizza for the same price?" Students work in small groups using a mathematical strategy of their choice to discover the answer. Students will also write a short paragraph indicating why it is better to buy either the two 9" pizzas or the one 14" pizza.
6. Go over the scoring rubric before students get into small groups.
7. Go over the problem with the class. Let the students explain on the board how they worked on the problem.
8. Show a visual of two 7-inch pizzas and one 14-inch pizza. Discuss the difference in the area.
9. Students go back into their small groups and do the following open-ended problem:
"Make an advertisement sign for a pizza company. Advertise a deal of 3 pizzas for a certain price versus 1 pizza for the same price. The one pizza **MUST** be the better deal. HINT: You are trying to fool the customer, just like I fooled you! Show any mathematical procedures that prove that the one pizza is a better deal than the three pizzas. Write in words the procedures that you used to come up with the sizes of the pizzas. Also, explain in words why your advertisement may seem misleading to the customers."
10. Remind students of the scoring rubric.
11. Go over the students' results. Students explain their answers to the open-ended problem and the mathematical procedures they used to find an answer.
12. Have a class discussion on what the students learned today and how they could use this knowledge in the real world.

STUDENT ASSESSMENT: Students can easily assess their own performance because they will have partners to discuss their thoughts and answers. I will observe the students while they are working in their

small groups. At the end of the lesson I will assess students' understanding by having them write a short paragraph explaining why it is better to purchase the 14" pizza. I will use the scoring guide when I look at the students' answers to their open-ended problem to further assess their learning.

Three Examples of Possible Solutions:

Solution 1:



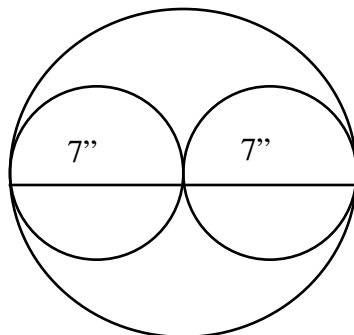
We have the best 4th of July deal:
Buy **THREE** 7" pizza's for \$12.00
Or
Buy one 14" pizza for \$12.00

The procedure we used: ~~We first decided that we wanted our one pizza to be 14".~~ Then we had to come up with the size for our three smaller pizzas. We knew we were trying to fool the customer, so the sum of the diameter of the smaller pizzas had to add up to more than 14". The first size that would work would be 5" pizza's because $5" + 5" + 5" = 15"$. Although, it is larger than 14", we didn't think it would be as convincing as 7" pizzas. If you add the diameter of three 7" pizza's you get $7" + 7" + 7" = 21"$. We thought this would be more misleading. We then found the area of one 14" pizza. It is equal to 153.86 in^2 . Next, we found the area for one 7" pizza. The area of one 7" pizza is equal to 38.465 in^2 . We then multiplied it by 3 because the customer gets three 7" pizzas. The total area for the three 7" pizzas is 115.395 in^2 . We used 3.14 for pi in all of our calculations. Our calculations proved that the one 14" pizza was a better deal than the three 7" pizzas.

Why may this seem misleading to the customer?

Our advertisement may seem misleading because most people would think that because you are buying three 7" pizzas that you are getting a total amount of pizza that is equal to a 21" pizza. Therefore they would think that 21" is a lot more than one 14" pizza. What the customers may not understand is how the area relates to the diameter. The diagram below proves why this does not work.

Here you can clearly see that one 14" pizza is much larger than two 7" pizzas.



Our Calculations:

Area for the three 7" pizzas: Area of a Circle = radius² x π
 π is approximately 3.14

$$\begin{aligned} \text{Area of one 7" pizza} &= (3.5")^2 \times 3.14 \\ &= 38.465 \text{ in}^2 \\ \text{Area of three 7" pizzas} &= 38.465 \text{ in}^2 \times 3 \\ &= 115.395 \text{ in}^2 \end{aligned}$$

Area for one 14" pizza: Area of a Circle = radius² x π
 π is approximately 3.14

$$\begin{aligned} \text{Area of one 14" pizza} &= (7")^2 \times 3.14 \\ &= 153.86 \text{ in}^2 \end{aligned}$$

Solution 2:

Buy **THREE**
6" by 5" pizzas
for only **\$15.00!!!**



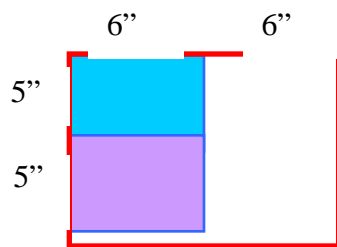
WE HAVE THE BEST DEALS IN TOWN!

Buy one
12" by 10" pizza
for **\$15.00**

The procedure we used: We decided that instead of using round pizzas we would try using rectangular pizzas. First we decide on the size of our large pizza. We decide on a 12" by 10" pizza. This would be like the Beast at Snappy Tomato. Then we decide that we could fool a customer by having our smaller pizzas be half the dimensions of our larger pizza. This would make our smaller pizzas 6" by 5". We then did the calculations to make sure the larger pizza would be the better deal. We found the area of the larger pizza by multiplying the length by the width. So the area would be 12" x 10" which equals 120 in². Then we found the area of the smaller pizza. The area of a rectangle is length times width again. So the area is 6" x 5" which equals 30 in². To find the area of the three 6" by 5" pizzas, you take the area of the one 6" by 5" pizza and times that answer by three. The total area for the three smaller pizzas is equal to 90 in². Our calculations proved that the 12" by 10" pizza equaled a larger area then the three 6" by 5" pizzas.

Why may this seem misleading to the customer?

Our advertisement may seem misleading because some people might think that when you have two 6" by 5" pizzas that it is equal to one 12" by 10" pizza. Therefore they would think that they are getting one extra 6" by 5" pizza. What the customer may not understand is that when you buy two of a certain size pizza, you are not doubling the dimensions of the pizza. The picture below shows why this does not work.



The blue area indicates the area of one 5" by 6" pizza. The purple area indicates the area of another 5" by 6 inch pizza. The red outline indicates the area of one 12" by 10" pizza. As you can see, it would take four 5" by 6" pizzas to equal one 12" by 10" pizza.

Our Calculations:

Area for the three 6" by 5" pizzas: Area of a Rectangle = length x width

$$\begin{aligned} \text{Area of one 6" by 5" pizza} &= 6" \times 5" \\ &= 30 \text{ in}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of three 6" by 5" pizzas} &= 30 \text{ in}^2 \times 3 \\ &= 90 \text{ in}^2 \end{aligned}$$

Area for one 14" pizza:

Area of a Rectangle = length x width

$$\begin{aligned} \text{Area of one 12" by 10" pizza} &= 12" \times 10" \\ &= 120 \text{ in}^2 \end{aligned}$$

Solution 3:



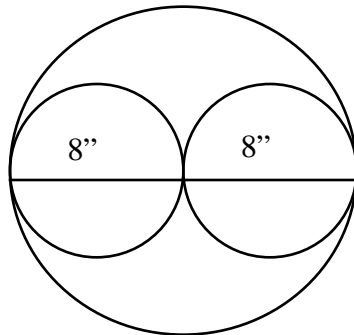
Buy THREE 9" pizzas for only \$18.00 or buy one 16" pizza for the same price.

~~The procedure we used: We used guess and check. We first decided that we wanted our large pizza to be 16". Then we had to come up with the size for our three smaller pizzas. We knew we were trying to fool the customer, so the sum of the diameter of the smaller pizzas had to add up to more than 16". The first size that would work would be 6" because 6" + 6" + 6" = 18". Although it is larger than 16", we didn't think it would be too convincing. We decided that we wanted to use the largest possible diameter for the smaller pizzas without going over the area of the larger pizza. We first found the area for the 16" pizza. The area for the 16" pizza was 200.96 in². Then the first diameter we tried for the smaller pizza was 10 inches. The area of one 10" pizza is 78.5 in². So the area of three 10" pizzas is 235.5 in². This is too large, so we tried 9" pizzas. The area for one 9" pizza is equal to 63.58 in². We then multiplied it by 3 because the customer gets three 9" pizzas. The total area of the three 9" pizzas is 190.755 in². We used 3.14 for pi in all of our calculations. Our calculations proved that the one 16" pizza was a better deal than the three 9" pizzas. This also showed us that if we use whole numbers for the diameter, than 9" is the largest we could make the diameter of the smaller pizzas.~~

Why may this seem misleading to the customer?

Our advertisement may seem misleading because most people think that because they are buying three 9" pizzas that they are getting a total amount of pizza that is equal to a 27" pizza. Therefore they would think that 27" is a lot more than one 16" pizza. The customers may not understand is how the area relates to the diameter. The diagram below proves why this does not work.

Here you can clearly see that one 16" pizza is much larger than two 8" pizzas.



Our Calculations:

Area of one 14" pizza:

Area of a Circle = radius² x p

p is approximately 3.14

$$\begin{aligned} \text{Area of one 16'' pizza} &= (8'')^2 \times 3.14 \\ &= 200.96 \text{ in}^2 \end{aligned}$$

Area of the three 10'' pizzas: Area of a Circle = radius² x π
π is approximately 3.14

$$\begin{aligned} \text{Area of one 10'' pizza} &= (5'')^2 \times 3.14 \\ &= 78.5 \text{ in}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of three 10'' pizzas} &= 78.5 \text{ in}^2 \times 3 \\ &= 235.5 \text{ in}^2 \end{aligned}$$

(This is larger than the area of the 16'' pizza, so the 10'' pizzas will not work.)

Area of the three 9'' pizzas: Area of a Circle = radius² x π
π is approximately 3.14

$$\begin{aligned} \text{Area of one 9''} &= (4.5'')^2 \times 3.14 \\ &= 63.585 \text{ in}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of three 9''} &= 63.585 \text{ in}^2 \times 3 \\ &= 190.755 \text{ in}^2 \end{aligned}$$

Scoring Rubric

Assessment Criteria	1	2	3
Depth of Understanding	Little understanding of area; many mathematical errors	Some understanding of area; few mathematical errors	In-depth understanding of area; well-developed explanations
Fluency	Incomplete approach or unworkable approach	A workable approach with little detail added	A workable approach with lots of detail added
Originality	Method leads to a solution, but is fairly common	Unusual method; this method is workable and only used by a few students	Unique method; this method is used only by one or two students
Elaboration or Elegance	Little explanation is given about procedures used to find area	Some explanation is given about procedures used to find area	In-depth clear, precise explanations are given about procedures used to find area; makes good use of graphs, charts, models, equations, etc.
Generalizations and Reasoning	Very few generalizations are made about why area is misleading to customers or incorrect reasoning	Some generalizations are made about why area is misleading to customers; supported with clear reasoning	Several well-supported generalizations about why area is misleading to customers; clear supported reasoning

Impact-Reflection/Analysis of Teaching and Learning:

Group Results

Assessment Criteria	1 (Novice)	2 (Proficient)	3 (Distinguished)
Depth of Understanding	Little understanding of area; many mathematical errors Number of Groups: 0	Some understanding of area; few mathematical errors Number of Groups: 4	In-depth understanding of area: well-developed explanations Number of Groups: 8
Fluency	Incomplete approach or unworkable approach Number of Groups: 2	A workable approach with little detail added Number of Groups: 8	A workable approach with lots of detail added Number of Groups: 2
Originality	Method leads to a solution, but is fairly common Number of Groups: 12	Unusual method; this method is workable and only used by a few students Number of Groups: 0	Unique method; this method is used only by one or two students Number of Groups: 0
Elaboration or Elegance 1 Group Scored a Zero	Little explanation is given about procedures used to find area Number of Groups: 2	Some explanation is given about procedures used to find area Number of Groups: 9	In-depth clear, precise explanations are given about procedures used to find area; makes good use of graphs, charts, models, equations, etc. Number of Groups: 0
Generalizations and Reasoning 1 Group Scored a Zero	Very few generalizations are made about why area is misleading to customers or incorrect reasoning Number of Groups: 3	Some generalizations are made about why area is misleading to customers; supported with clear reasoning Number of Groups: 6	Several well-supported generalizations about why area is misleading to customers; clear supported reasoning Number of Groups: 2

The table above is the scoring rubric I used for the open-ended responses. I have indicated in red the number of groups that scored at each level. The scoring rubric analysis helped me see the success of instruction for this lesson. The data shows that the majority of students understood area and could compute area accurately. A majority of the students could also explain the procedures to find area of circles and they could make generalizations about why area is misleading to some customers.

There is one major problem I noticed as I looked over the results from the rubric. I had all twelve groups score at the novice level for Originality. The students had a variety of answers, but all the groups approached the problem in the same manner. Most groups used the Guess and Check strategy.

The objective of this lesson was, “Students will be able to use mathematical strategies to compare area of circles when given the diameter.” Looking at the data above, I believe most students met the objective. There were twelve groups that responded to the open-ended problem. Each group had two or three students. Eight of the twelve groups scored at the highest level in the Depth of Understanding category. This means that two-thirds of the students completed calculations for area with virtually no errors. Four groups scored at the Proficient level. This means that one-third of the students understood how to calculate area for circles, but had a few mathematical errors.

*** An in-depth analysis and examples of student work is at the end of the lesson.***

Refinement-Lesson Extension/Follow-up:

As shown in my reflection most of my students reached the objective for this lesson, so I do not think I would have subsequent lessons to reinforce these concepts. Since it was the end of the year, I did not have time to extend on these concepts. When I teach this lesson next year, I have some ideas for ways to extend the concepts learned in this lesson. Students could explore how the diameter affects the area. I would have the students look at the circles that have a certain diameter and look how the area changes when you double the diameter or triple the diameter. I would ask them, "If you double the diameter of a circle, does this double the area?" The students could also explore how circumference and area of circles are related.

There are a few things I would change if I taught this same lesson again. I would first split the lesson into two lessons. It was really hard to get through everything in one class period. During the first lesson I would keep procedures 1 – 7. For procedure 8, I would have the students construct the pizza models, instead of pre-making the models myself. I think this would help the students visualize why two 7" pizzas do not have the same area as one 14" pizza. During the second lesson, I would only do the open-ended problem. This would give the students more time to try different ways to solve the problem and it would also give them more time to share. I would also stress the Originality category on the scoring rubric.

After the students had a good understanding on circumference and area of circles, I would introduce other concepts that built upon the concepts of area. The next concept I would teach them is surface area. The students would discover ways to find the surface area for 3-dimensional shapes such as cones and cylinders. Another concept that I could teach them is volume. Since they have already learned about area, this would help them understand how to find the volume for 3-dimensional figures.

Analysis of Satisfactory Student Work

Scoring Rubric for Group 1 (Highlighted in Yellow is what they scored)

Assessment Criteria	1	2	3
Depth of Understanding	Little understanding of area; many mathematical errors	Some understanding of area; few mathematical errors	In-depth understanding of area; well-developed explanations
Fluency	Incomplete approach or unworkable approach	A workable approach with little detail added	A workable approach with lots of detail added
Originality	Method leads to a solution, but is fairly common	Unusual method; this method is workable and only used by a few students	Unique method; this method is used only by one or two students
Elaboration or Elegance	Little explanation is given about procedures used to find area	Some explanation is given about procedures used to find area	In-depth clear, precise explanations are given about procedures used to find area; makes good use of graphs, charts, models, equations, etc.
Generalizations and Reasoning	Very few generalizations are made about why area is misleading to customers or incorrect reasoning	Some generalizations are made about why area is misleading to customers; supported with clear reasoning	Several well-supported generalizations about why area is misleading to customers; clear supported reasoning

This group did a very nice job on their open-ended problem. I did let the groups do their advertisement signs at home. This was the only group that added more detail at home to their explanation on how to find area and they also added more detail to their generalizations on why area is misleading to customers.

The rubric above shows how I scored this group. I will explain in-depth why I evaluated their work the way I did for each category.

Depth of Understanding: This group had the correct calculations. I could tell that they knew how to calculate area. The only problem I found was the labeling. This group forgot to label some things and the also wrote “ 2” ” for squared inches.

Fluency: This group scored a three because they had a good approach to find the area of the circles and they showed all of their calculations. They also explained their calculations well.

Originality: This group scored a one because all the groups attempted this problem in the same way. All the groups just picked diameters that they thought would work and then they tested them to make sure the diameters would work.

Elaboration or Elegance: I like the fact that this group gave a general formula for finding the area of a circle. I gave this group a two because in their explanations they forgot to square the radius when find the area. I had a lot of groups forget that when they explained their process in words.

Generalizations and Reasoning: This group scored a three in this category because they expanded on why their advertisement is misleading to the customer and they explained in more detail what actually is true about

the area of the pizzas. They discussed what the customer would actually think they were getting and they also discussed the actual difference between the two areas.

Analysis of Unsatisfactory Student Work

Scoring Rubric for Group 2 (Highlighted in Yellow is what they scored)

Assessment Criteria	1	2	3
Depth of Understanding	Little understanding of area; many mathematical errors	Some understanding of area; few mathematical errors	In-depth understanding of area; well-developed explanations
Fluency	Incomplete approach or unworkable approach	A workable approach with little detail added	A workable approach with lots of detail added
Originality	Method leads to a solution, but is fairly common	Unusual method; this method is workable and only used by a few students	Unique method; this method is used only by one or two students
Elaboration or Elegance +0 (No Work)	Little explanation is given about procedures used to find area	Some explanation is given about procedures used to find area	In-depth clear, precise explanations are given about procedures used to find area; makes good use of graphs, charts, models, equations, etc.
Generalizations and Reasoning +0 (No Work)	Very few generalizations are made about why area is misleading to customers or incorrect reasoning	Some generalizations are made about why area is misleading to customers; supported with clear reasoning	Several well-supported generalizations about why area is misleading to customers; clear supported reasoning

This group did not do well on this problem. They did not answer all the parts to the problem. They did try to complete some calculations and an advertisement sign, but they did not explain their procedures in words or why their advertisement may seem misleading to customers.

The rubric above shows how I scored this group. I will explain in-depth why I evaluated their work they way I did for each category.

Depth of Understanding: This is the only category in which this group scored a two. The reason this group earned a two is because they did come up with the total area for one 15” pizza and three 6” pizzas. These pizza sizes will work for this problem. I believe this group used a calculator to find the area for the 15” and 6” pizzas.

Fluency: The reason this group scored a one is because their calculations at the top of their paper showed incorrect procedures. It looked like they were trying to find the area for three 8” pizzas and one 20” pizza. This group forgot to square the radius when doing their calculations, therefore the total area for the pizzas was incorrect. The also did not show any work for the 15” pizza and the three 6” pizzas.

Originality: This group scored a one for the same reasons as I did for group one.

Elaboration or Elegance & Generalization / Reasoning: This group scored a zero because they did not attempt this part of the open-ended problem.