

## **Title: Binostat Project**

**Name:** Alice Gabbard    **Date Scheduled:** 11/5-7/03    **Lesson Length:** 3 hours  
**School:** NMS    **Grade:** 7<sup>th</sup>    **# of Students:** 22  
**Subject:** Math    **Topic:** Probability/  
Fractions/Algebra    **# of IEP's:** 7

### **Objective:**

Students will complete the first 12 rows of Pascal's Triangle and play a game, which uses the Binostat Ball Drop to enact the number pattern in Pascal's Triangle. Students will use the Thinking Rubric to guide their responses to think deeply while analyzing the theoretical probability of the game.

### **Big Ideas:**

#### *Probability*

PS.7.7—Determine theoretical (mathematical) probabilities, compare to experimental results, and explain reasons why there might be differences.

#### *Algebraic Ideas*

A.7.1-Recognize, create, and continue *patterns*, giving the generalization for the *n*th term. Use variables to describe numerical patterns.

### **Targeted Standards:**

#### Kentucky Core Content for Mathematics

- MA-M-3.2.4 Calculate theoretical probabilities and tabulate experimental results from simulations
- MA-M-3.2.5 Make predictions and draw conclusions from statistical data and probability experiments
- MA-M-3.2.6 Use counting techniques, tree diagrams, area models, and tables to solve probability problems
- MA-M-3.3.2 How theoretical probability and experimental probability are related

#### Kentucky Program of Studies

- M-7-PS-2: Students will make predictions, draw conclusions, and verify results from statistical data and probability experiments.
- M-7-PS-9: Students will determine theoretical (mathematical) probabilities, compare to experimental results, and explain reasons why there might be differences, (e.g., express probability as a ratio, decimal, or a percent as appropriate for a given situation).
- M-7-A-1: Students will recognize, create, and continue patterns and generalize the pattern by giving the rule for any term.

### **Connections:**

Students will explore the number patterns in Pascal's Triangle and discover connections between that number sequence and the theoretical probabilities in the Binostat game. Students will also discover the powers of two found in the row totals of Pascal's Triangle and in the denominators of the probabilities.

**Context:**

Having explored exponential growth (doubling), fraction concepts, fraction addition, and simple probability, this rich learning task will provide an opportunity for students to apply those skills and concepts to in-depth study and extend their learning through self-direction as they are given an opportunity to “Think like mathematicians.” Eventually they will solve other problems related to Pascal’s Triangle, such as counting pathways between two places, and finding the probabilities of flipping coins.

**Timeline:** November 5—7, 2003

**Materials/Resources Needed:**

Binostat Ball Drop  
Binostat Project Handout  
Thinking Rubric  
Binostat Drawing

**Technology Needed:**

Calculators

Pascal’s Triangle Worksheet:

<http://mathforum.org/workshops/usi/pascal/images/midd.comb1.gif>

**Lesson/Procedure:**

1. Ask students to review the Thinking Rubric and remind them to “Think Like Mathematicians;” “Think deeply about simple things;” and to “Question the answers, don’t just answer the questions.”
2. Students will investigate the number pattern in the first 4 rows of Pascal’s Triangle and complete the next 8 rows by adding two adjacent cells together and writing the sum underneath. Students will also find the sum for each row.
3. Students will write a description/generalization of 5 patterns they observe in the triangle.
4. Discuss the rules of the Binostat game: Each player (in the pair) drops 5 balls for each stage and then records the scores formed by all 10 balls, according to the scoring chart. The player with the highest score after playing all 6 stages is the winner.
5. Ask each player to write a prediction and explanation of which player he or she thinks will win.
6. Have the students arrange themselves in pairs. The student in each pair with next birthday should choose which player he or she would like to be.
7. Set the bottom row at stage 2. Allow students to take turns dropping 10 balls per stage and recording the scores. (I have one Binostat for the entire class, so each pair of students will have to use the same score).
8. Ask students to explore the outcomes by counting and recording the number of ways a ball could travel to each finish space.

Alice Gabbard

[agabbard@newport.k12.ky.us](mailto:agabbard@newport.k12.ky.us)

9. Ask students to find and record the total number of ways to reach all the finish spaces in each stage.
10. Ask students to find and record the probability of the ball landing in each finish space for each stage.
11. Ask students to determine if the game is fair by finding the theoretical probability of player A or player B winning the game. Compare these results to the experimental probability.
12. Ask students to think creatively by writing and exploring at least one additional question.

**Alterations for Individual Student Needs** (i.e. IEP, Gifted and Talented, etc.):

Visual models;

Manipulatives;

Assignment modification—fewer stages of the problem;

Cooperative group work;

Individual assistance;

Differentiated instruction through open-ended problem solving—a variety of responses at different levels will be valued.

**Assessment Type:**

Binostat Project/Rich Learning Task (Rubric Attached)

**Reflections for the Future:**

*Think Like a Mathematician*

*Think Deeply About Simple Things*

*Question the Answers, Don't Just Answer the Questions*

*The Real Mathematics Begins After the Original Question is Answered*

### Scoring Rubric to Encourage Thinking Like a Mathematician

<i>Assessment Criteria</i>	<b>1 Novice</b>	<b>2 Apprentice</b>	<b>3 Proficient</b>	<b>4 Distinguished</b>
<b>Depth of Understanding</b>	Little or no understanding	Partial understanding; minor mathematical errors	Good understanding; mathematically correct	In-depth understanding; well-developed ideas
<b>Fluency</b>	One incomplete or unworkable strategy or technique	At least one appropriate solution with strategy or technique shown.	At least two appropriate solutions using the same strategy or technique	Several appropriate solutions using the same strategy or technique
<b>Flexibility</b>	No method apparent	At least one method (e. g., all graphs, all algebraic equations and so on)	At least two methods of solution (e. g., geometric, graphical, algebraic, physical modeling)	Three or more methods of solution (e. g., geometric, graphical, algebraic, physical modeling)
<b>Originality</b>	Method may be different but does not lead to a solution	Method will lead to a solution but is fairly common	Unusual, workable method used by only a few students	Unique, insightful method used only by one or two students
<b>Elaboration or Elegance</b>	Little or no appropriate explanation given	Explanation is understandable but is unclear in some places	Clear explanation using correct mathematical terms	Clear, concise, precise explanations making good use of graphs, charts, models, or equations
<b>Generalizations and Reasoning</b>	No generalizations made, or they are incorrect and reasoning is unclear	At least one correct generalizations made; but not well-supported with clear reasoning	At least one well-made, supported generalization, or more than one correct but unsupported generalization	Several well-supported generalizations; clear reasoning
<b>Extensions</b>	No related mathematical question explored	At least one related mathematical question appropriately explored	One related question explored in-depth, or more than one question appropriately explored	More than one related question explored in-depth

*Mathematical Questions: What? What if? When? Where? Why? Why Not? How?*

**Binostat Project**  
*A Rich-Learning Task*

**Part 1—Pascal's Triangle**

- a. Complete the first 12 rows of Pascal's Triangle (see attached).
- b. Write the sum of each row on the right side of the attached page.
- c. Describe here 5 different patterns you find in Pascal's Triangle. Use the row numbers (starting with row 0 as the first line with only 1 cell; row 1 has 2 cells) to generalize any formulas you find.

Pattern 1	
Pattern 2	
Pattern 3	
Pattern 4	
Pattern 5	

## Part 2—The Binostat Game (The Experiment)

1. Write a prediction (hypothesis) of which player you think will most likely win the game by earning the most points in their finish spaces. Explain your reasoning.
2. The player with the next upcoming birth date chooses which player they want to be. Play the game using the Binostat Ball Drop and record the scores according to the number of balls that drop into each finish space for each stage.

Stage 1:

A	B
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Stage 2:

A	B	A
---	---	---

Stage 3:

A	B	B	A
---	---	---	---

Stage 4:

A	A	B	A	A
---	---	---	---	---

Stage 5:

A	A	B	B	A	A
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Stage 6:

A	A	B	B	B	A	A
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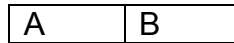
Total Scores: *Player A* \_\_\_\_\_ *Player B* \_\_\_\_\_

3. Compare the results of the game/experiment with the prediction you wrote above in number 1.
4. Revise your prediction based on the results of the game/experiment.

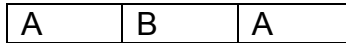
### Part 3—Binostat Game Analysis

- Find the number of pathways to reach each finish space.

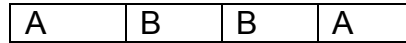
Stage 1:



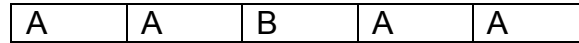
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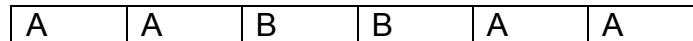
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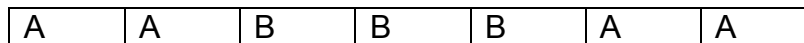
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Stage 5:

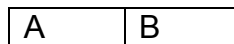


Stage 6:

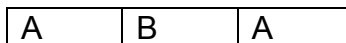


- Find the total number of pathways in each stage by adding the pathways to all the finish spaces in each stage. *Write the sum of the pathways to the right of the cells above.*
- Write the theoretical probability (fractional form) of a ball landing in each finish space.

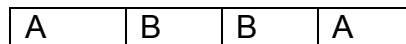
Stage 1:



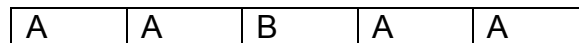
Stage 2:



Stage 3:



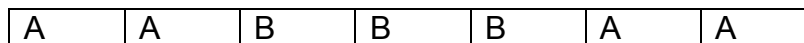
Stage 4:



Stage 5:



Stage 6:



- Is the game fair? Determine if the game is fair by finding the probability of player A winning the game and player B winning the game. Do the experimental results from the game played in part 2 confirm your conclusions? Use additional pages of paper to fully explore your mathematical ideas by showing a *variety* of methods, strategies, and solutions.
- Be creative! Write and explore at least one additional question related to this problem.