Optimizing Egg Revenue in Manitoba

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Abstract

Randy Wiebe was kind enough to introduce me to the chicken business he runs in Manitoba, Canada. He has 40,000 layers, which he ordinary swaps out about every 70 weeks.

It occurred to me that his "70 week" rule might not be optimal, given his data, and so I proposed that we try to discover the appropriate amount of time. This paper represents a preliminary model for the optimal life of a flock of hens.

1 Introduction

My friend Randy Wiebe is a chickenboy, riding herd over a flock of 40,000 hens in southern Manitoba, not far from Winnipeg.

As layers age, they become less productive: their eggs either don't come, or aren't what they used to be. This sad fact accounts for the chicken massacres that occur on a regular basis. At what point do we throw our older chicken over for some new, young, trophy hens? Since individual hen production is not monitored, the entire flock gets an up or down vote on whether they are permitted to live another day.

Generally the rule of thumb is 70 weeks for the type of hen that Randy raises (todo: name, variety, citation). So the date of their execution is known well in advance. Randy receives weekly egg production data for his flock from the egg grading facility in Winnipeg, which he monitors; but Randy hasn't used the data for the purposes of any analysis.

Our goal in this paper is to use the data and some reasonable estimates he can provide to determine if he has chosen the right number of weeks for his layer production and destruction. In spite of a dreary existence, it seemed to me when I looked at the data that the old girls are shown the door before they'd really exhausted their economic usefulness - perhaps I can convince Randy to spare their lives for a few more weeks.

There are some additional constraints that we should be aware of:

- when the layers are "swapped out" (to use a particularly benign euphemism for massacred), there is a one-week period when production goes to zero, because the hen houses are cleaned before the new birds arrive.

- In addition, Randy must **pay** for the massacre, because the hens can't be sold (simply because there's no market).

- Randy must also purchase the new birds, which cuts into his revenue.

So at every transition between flocks there are fixed costs. Here are the functions and parameters to be used in the following analysis:

- F - the fixed cost of transition from flock to flock.
- $p(t)$ - the weekly payout function for a given flock (the amount earned each week for the eggs). We're going to assume that this is constant for each flock. In reality, it varies, and, ideally, we'd be able to tell Randy whether to hold on to some keeper birds, or send some non-productive hens to the guillotine a little early.
- $R(t)$ - the total revenue function: the amount Randy is putting into the bank as a function of time t.
- T - the period of time we keep the layers in the house (also used for the optimal period itself).
- $A(T)$ - the average revenue function as a function of the period T.

2 Analysis

Now because we're assuming that the payment profile is the same for every flock, the optimal solution should be periodic. At time t, for period T, you will have already managed $N \equiv Floor(t/T)$ flocks, and you will be working on flock number $N+1$. Then

$$R(t) = \left[\int_0^T p(x)dx\right] \cdot N - F \cdot N + \int_0^{t-T\cdot N} p(x)dx$$

This expression for $R(t)$ illustrates the various inputs and outputs for the revenue stream: the first piece is the revenue from the N flocks already managed from time 0 to t; the second piece is for the N fixed cost expenses from time 0 to t; and the last piece is the amount obtained, so far, from the current flock.

We can simplify $R(t)$ a little:

$$R(t) = \left[\int_0^T p(x)dx - F\right] \cdot N + \int_0^{t-T\cdot N} p(x)dx$$

Now Randy's in this business for the long-haul, so he's interested in maximizing his **average** income, and over the foreseeable future. That suggests that we should maximize the "lifetime average",

$$\lim_{t\to\infty} \frac{R(t)}{t} = \lim_{t\to\infty} \left[\int_0^T p(x)dx - F\right] \cdot N + 0,$$

the transitory "current flock" term vanishing in the limit. Now

$$\lim_{t\to\infty} N = \lim_{t\to\infty} \frac{Floor(t/T)}{t} = \frac{1}{T}$$

so we obtain for our lifetime average revenue function the function

$$A(T) \equiv \lim_{t\to\infty} \frac{R(t)}{t} = \frac{1}{T}\left[\int_0^T p(x)dx - F\right].$$

Our job is to find the optimal value of the period, T, to maximize the lifetime average function. Most calculus students would know what to do: differentiate the function A with respect to T, and solve for a root of the derivative:

$$\frac{dA}{dT} = \frac{p(T)}{T} - \frac{\int_0^T p(x)dx - F}{T^2}$$

Setting this expression equal to zero, and simplifying a little, we obtain

$$0 = p(T) \cdot T - \left[\int_0^T p(x)dx - F\right]$$

We can now use a scheme such as Newton's method to discover values of T making this true, and then check to make sure that they're maxima, rather than minima.

Newton's method is an iterative scheme for finding roots. The general form for the iterates is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

In our case, with our function $f(T) = p(T) \cdot T - \left[\int_0^T p(x)dx - F\right]$, we obtain for the iterative scheme that

$$T_{n+1} = T_n - \frac{T_n \cdot p(T_n) - \left[\int_0^{T_n} p(x)dx - F\right]}{T_n \cdot p'(T_n)}$$

3 Results

3.1 An Example

To perform a test of this, I chose a sample form for $p(t)$, a normal density distribution of the form

$$p(t) = e^{-\left(\frac{t-35}{35}\right)^2}.$$

I set the fixed cost $F = 5$, so that we will see a revenue stream function $R(t)$ as in Figure 1.

After running Newton's method on this particular example, I found that there is an optimal period of approximately $T = 55.4$ weeks for the hens: the graph of $A(T)$ (Figure illustrates the sensitivity about the optimal value.

3.2 The Show: Using Data from Manitoba

The first things we must do are indicated above:

4 Appendix

4.1 Code

Here is a snippet of the code I used to create this example.

4.2 Figures

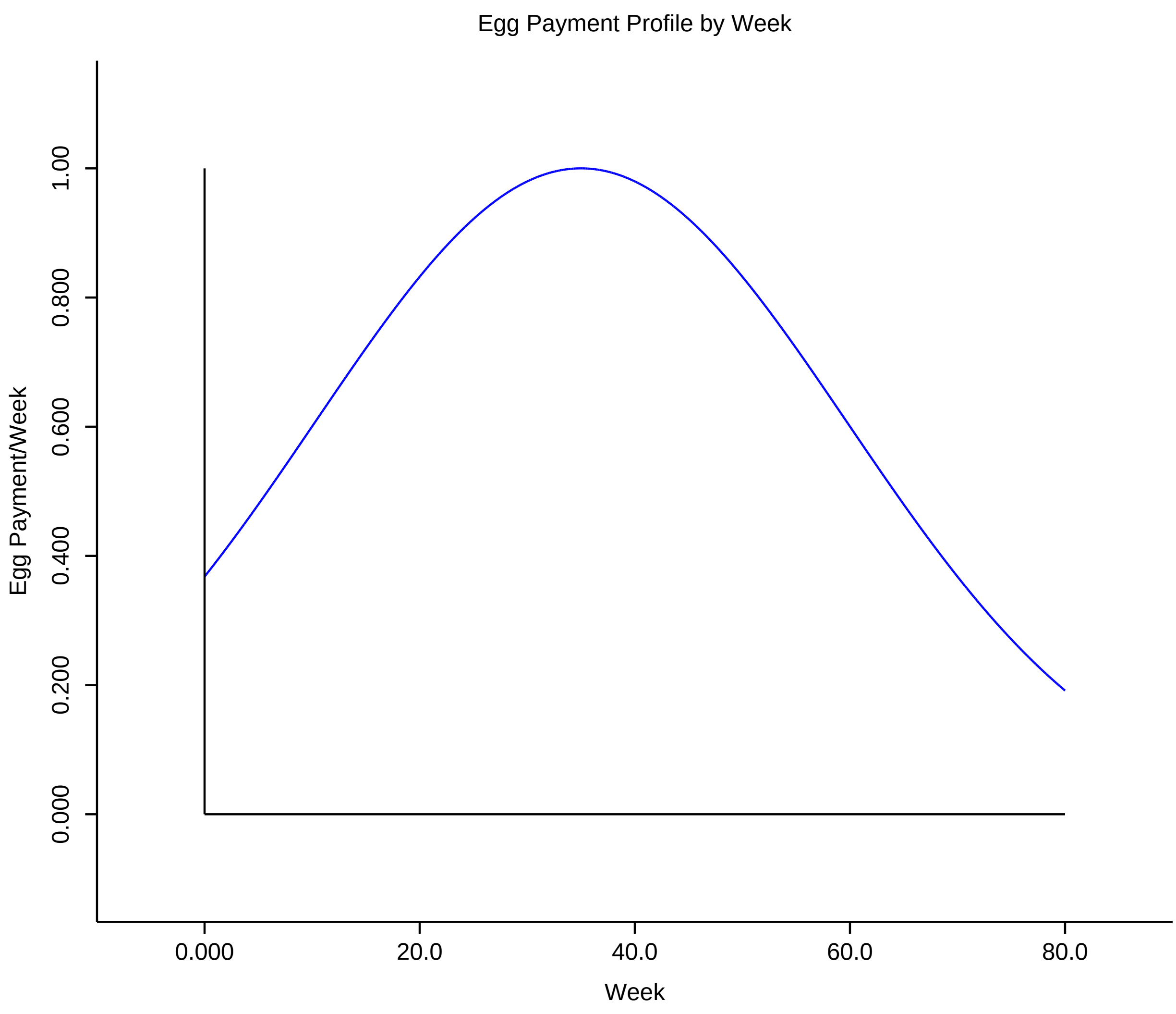

Figure 1: $p(t)$ - the weekly payout function for a given flock.

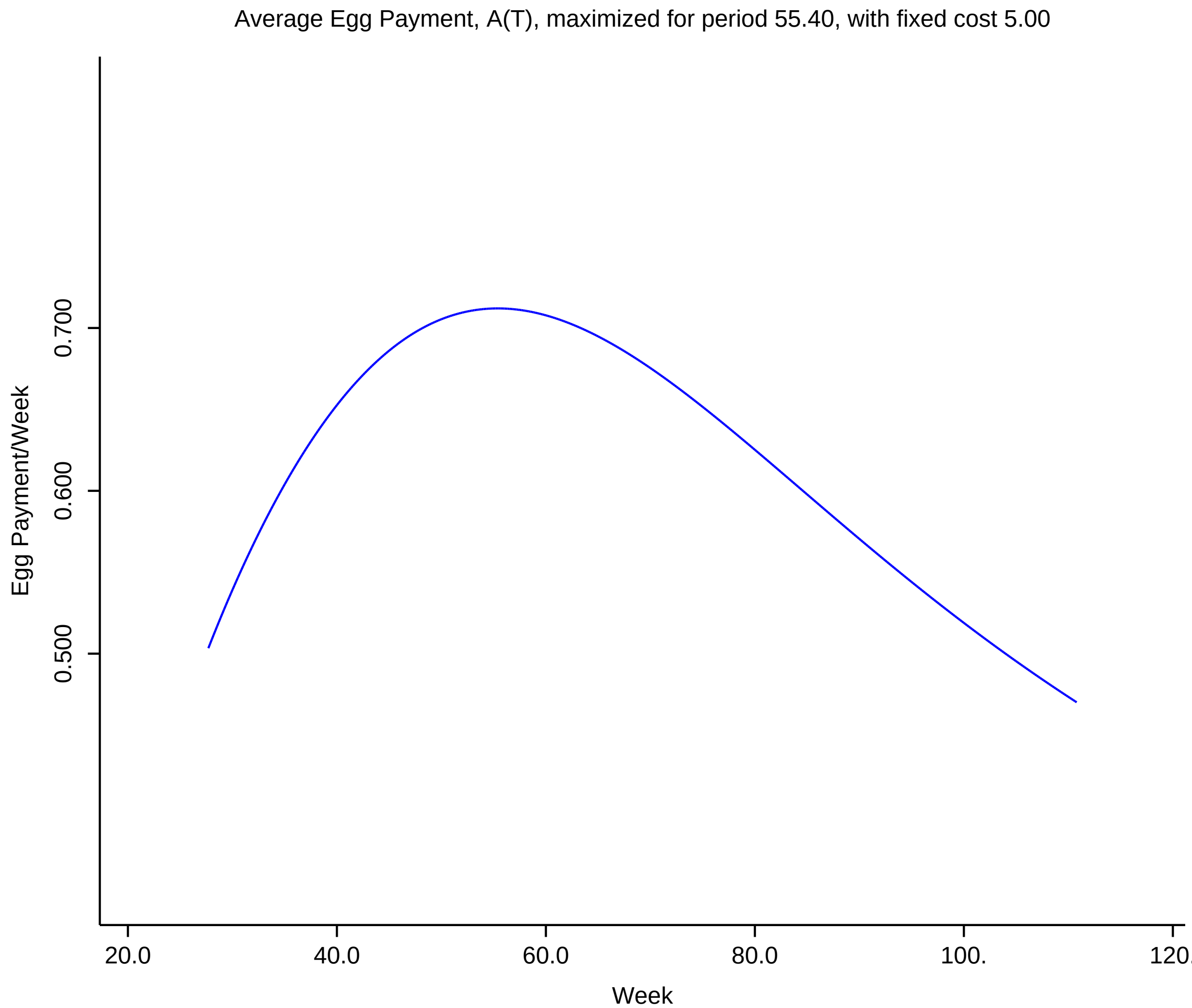

Figure 2: $A(T)$, the average revenue function, as a function of the period T. The maximum occurs at a value of $T = 55.4$ for this choice of $p(t)$ and F.

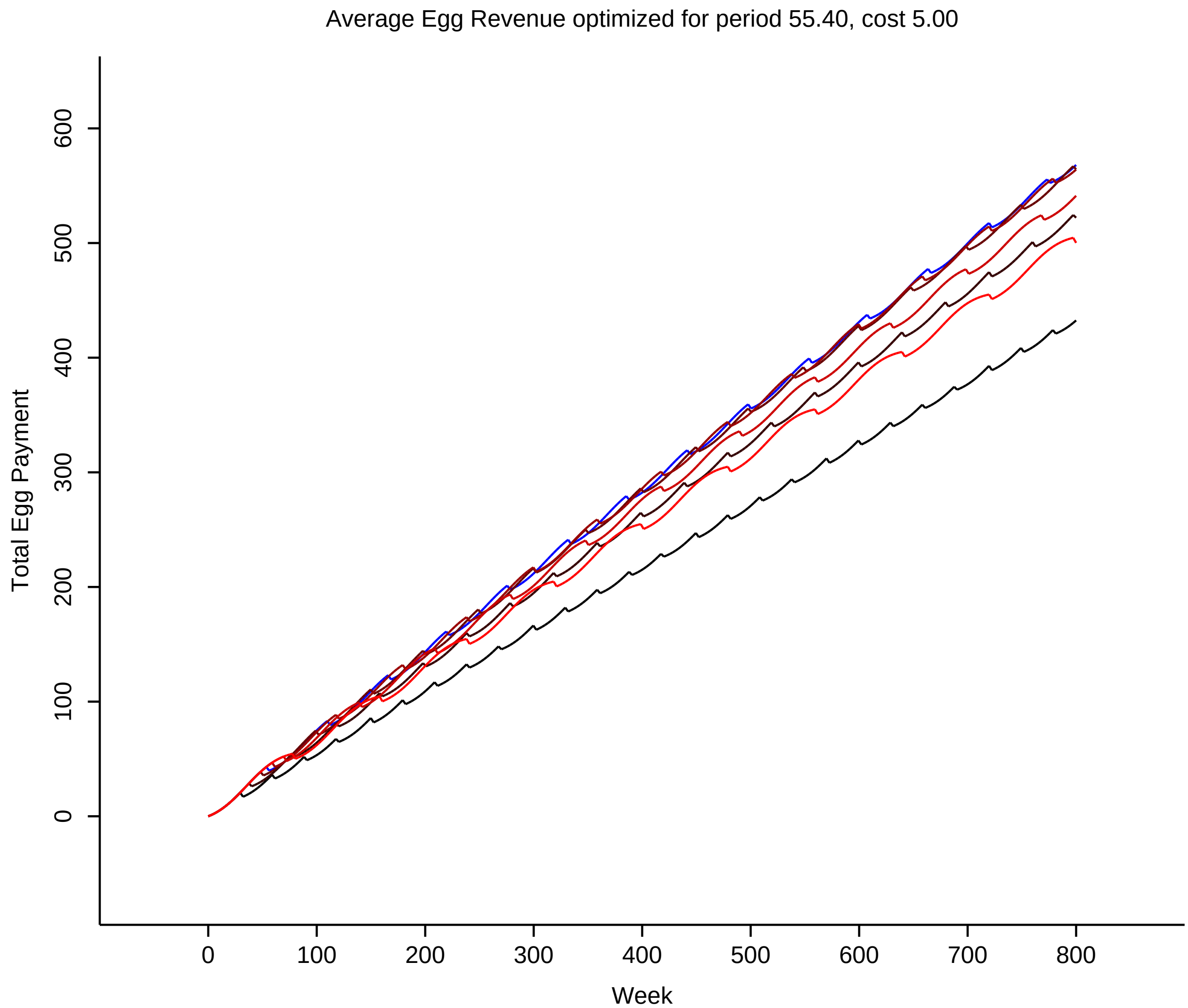

Figure 3: The blue curve is for optimal period; the darker the red, the smaller the period. Additional periods included are 30, 40, 50, 60, 70, and 80.

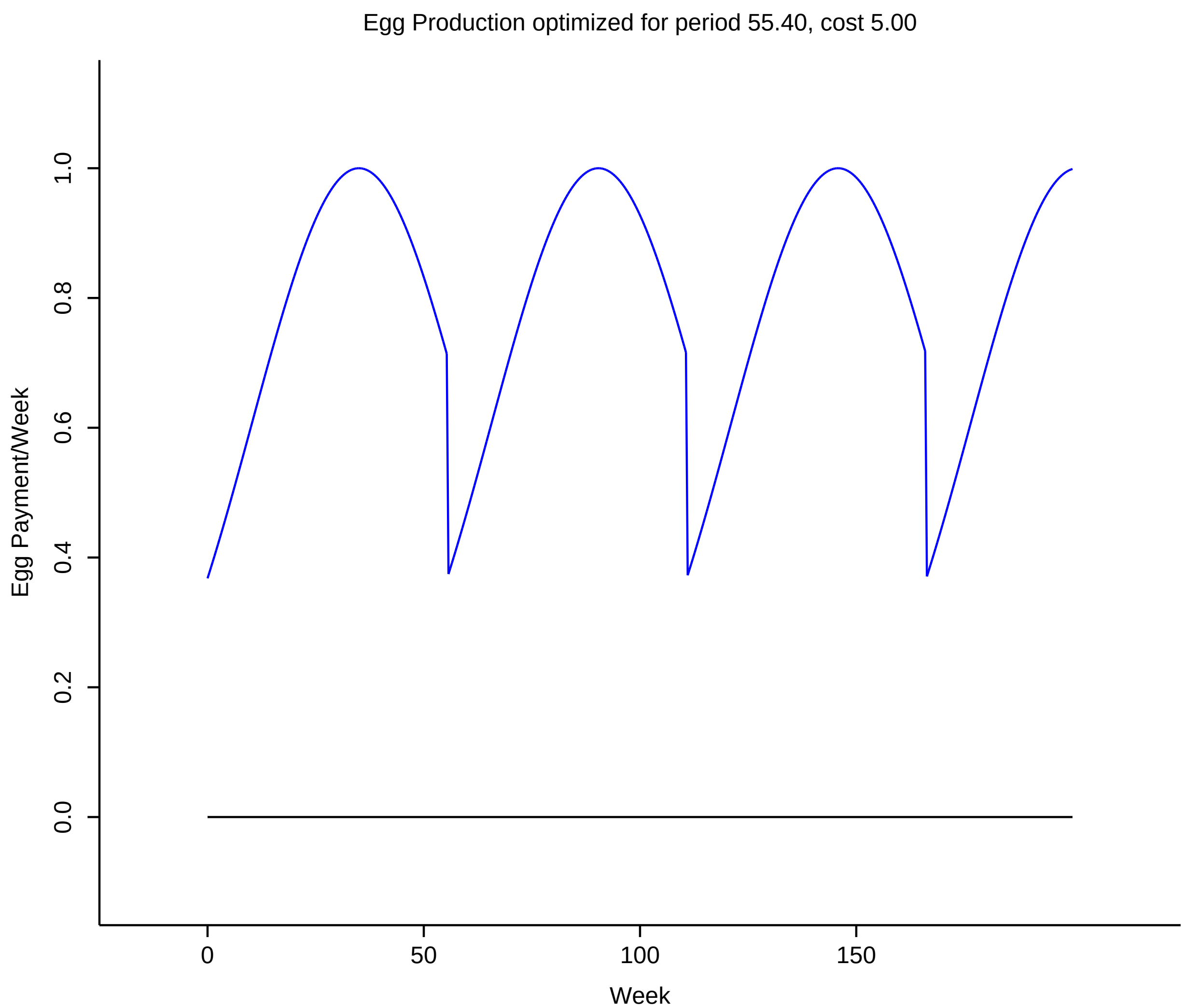

Figure 4: Given the choice of $p(t)$, the weekly payout function, and F, the fixed cost of transition, this is egg payout schedule (ignoring the down-time between flocks).

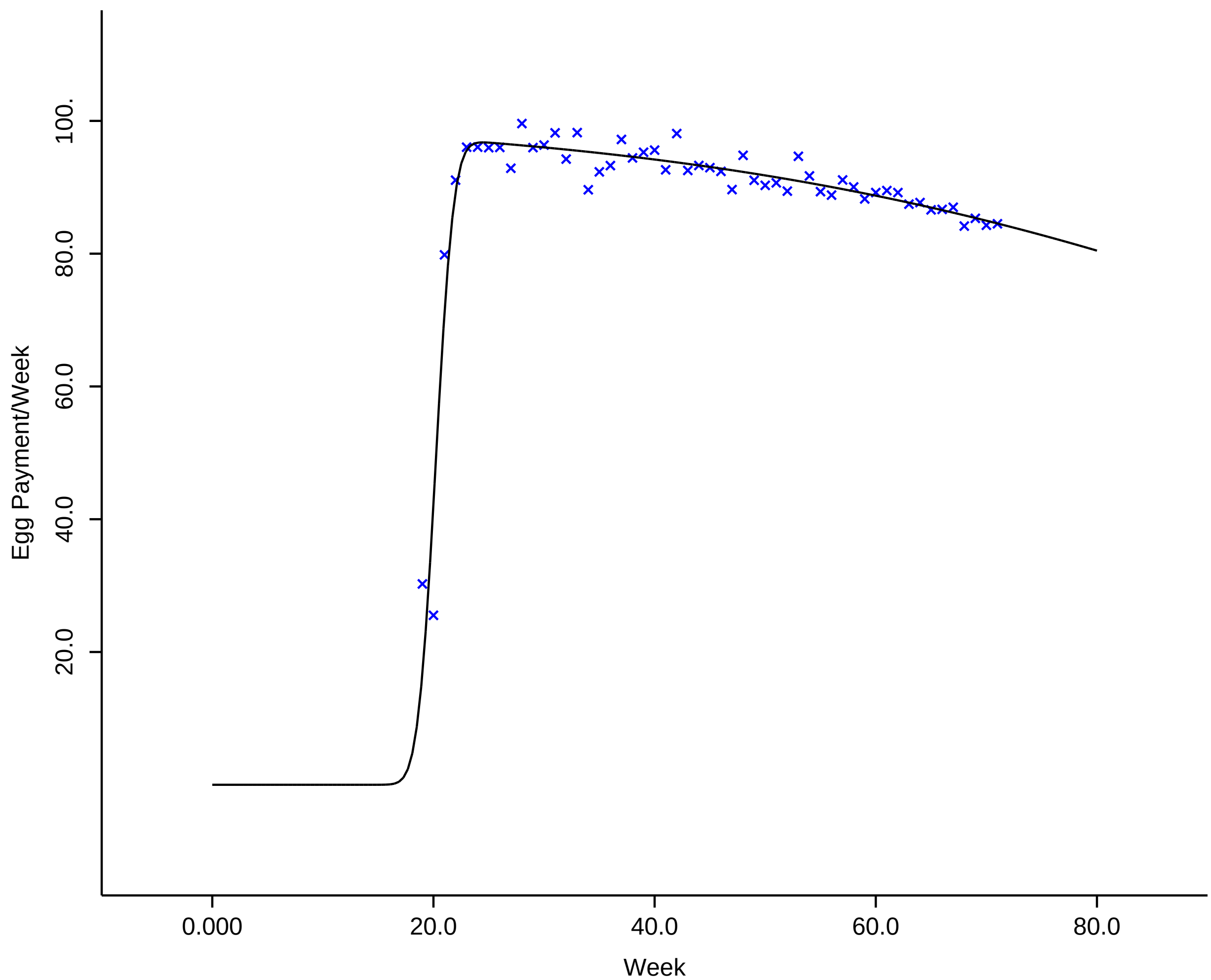

Figure 5: Data and a fit to the data

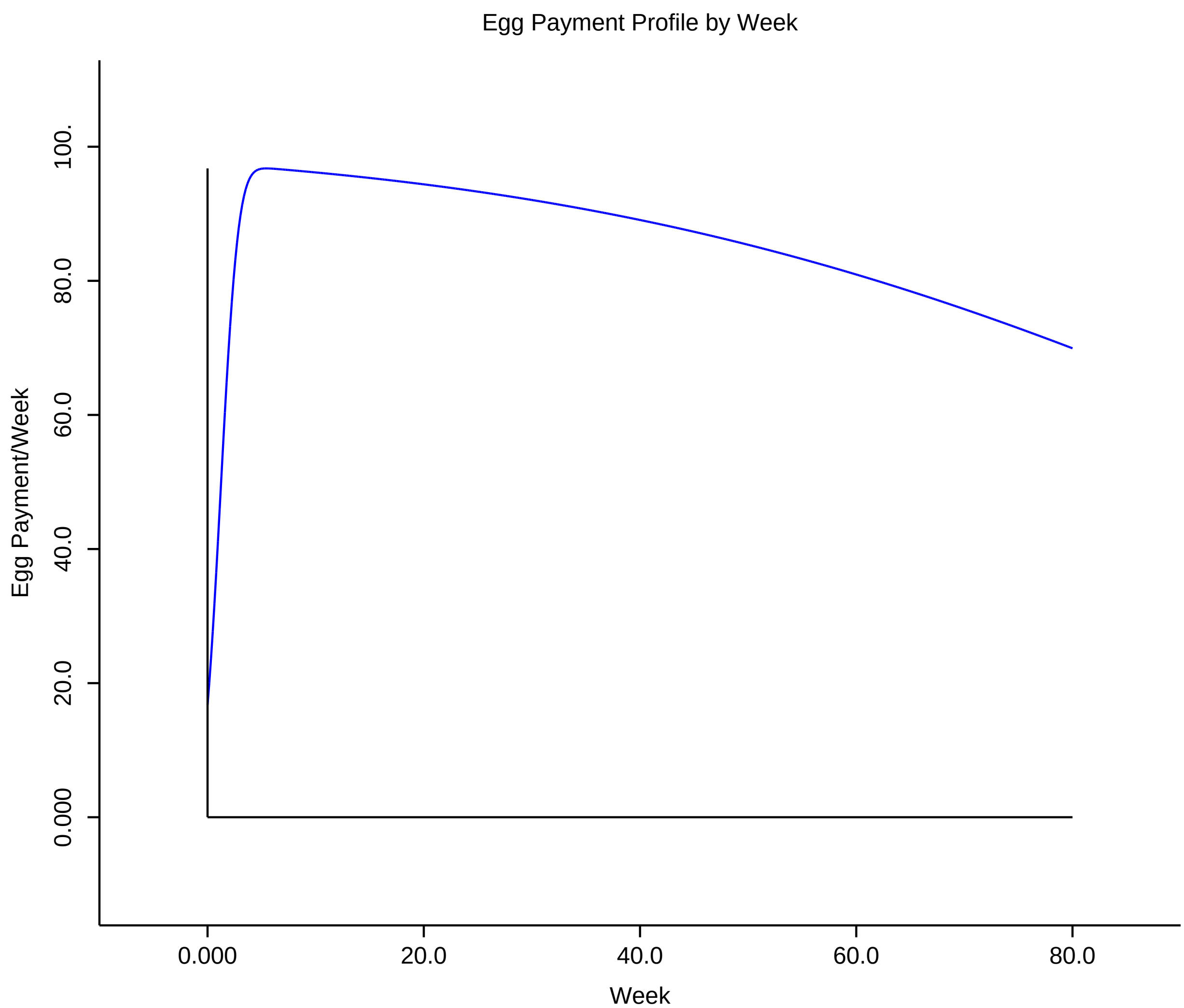

Figure 6: $p(t)$ - the weekly payout function for a given flock.

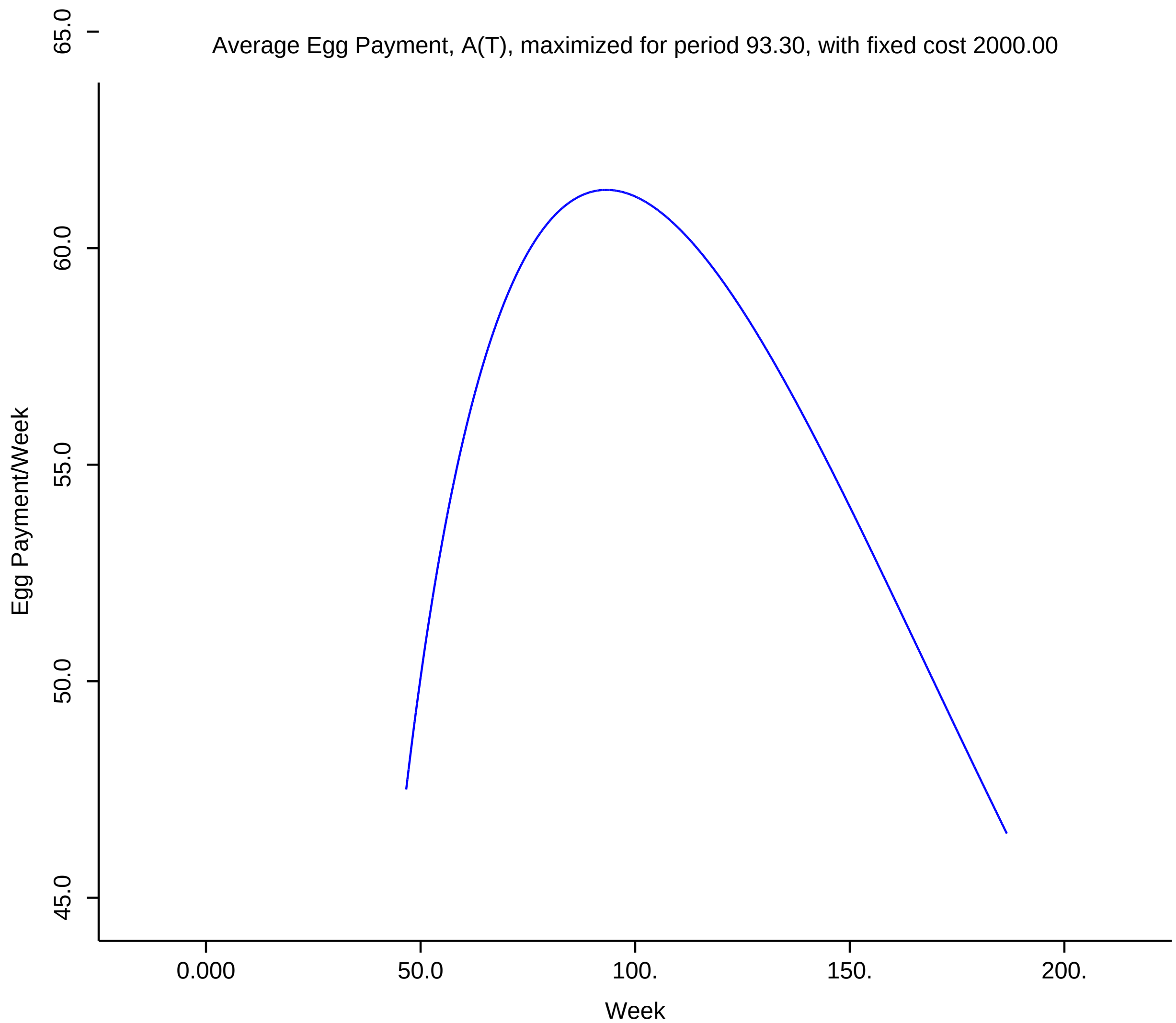

Figure 7: $A(T)$, the average revenue function, as a function of the period T. The maximum occurs at a value of $T = 55.4$ for this choice of $p(t)$ and F.

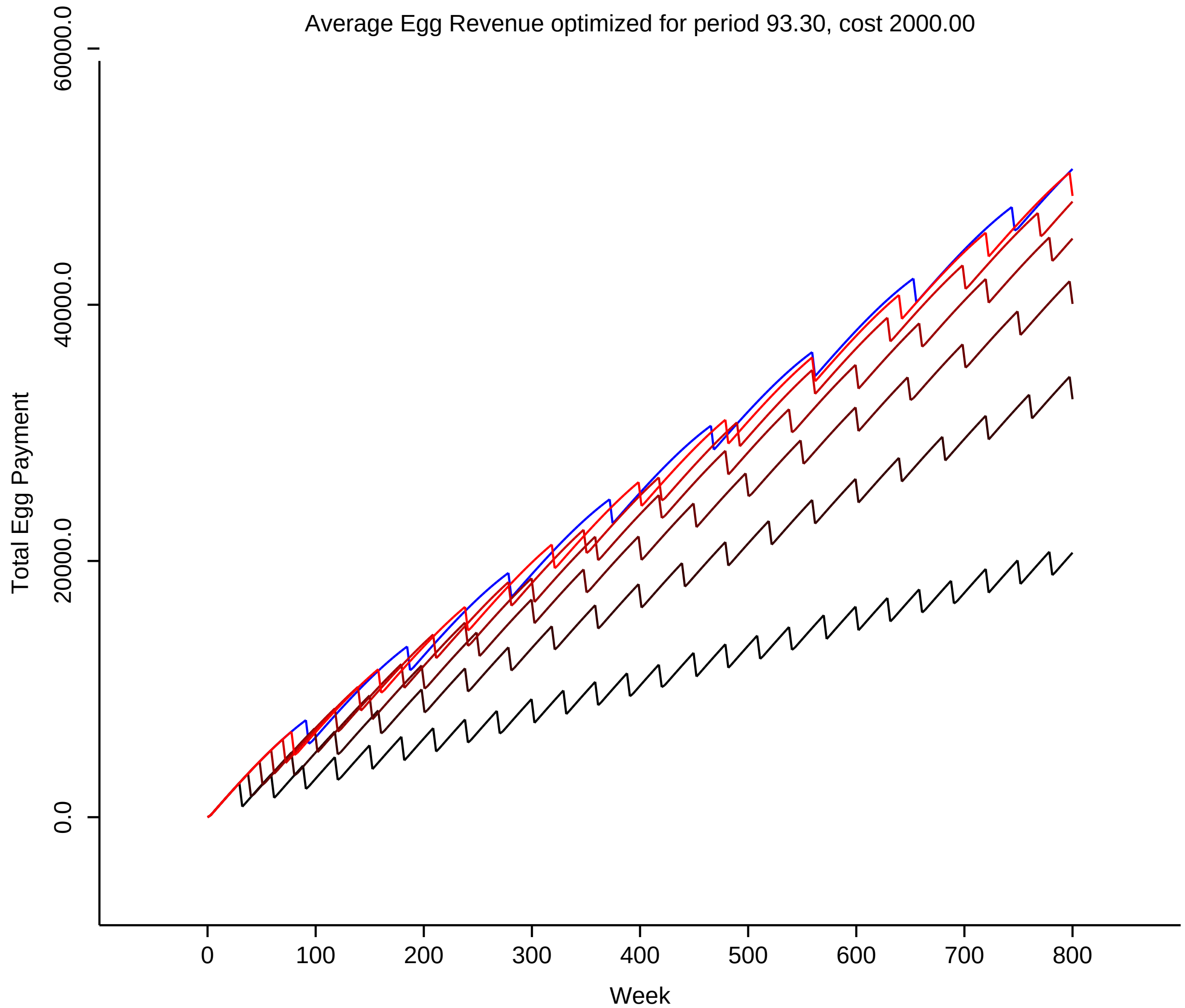

Figure 8: The blue curve is for optimal period; the darker the red, the smaller the period. Additional periods included are 30, 40, 50, 60, 70, and 80.

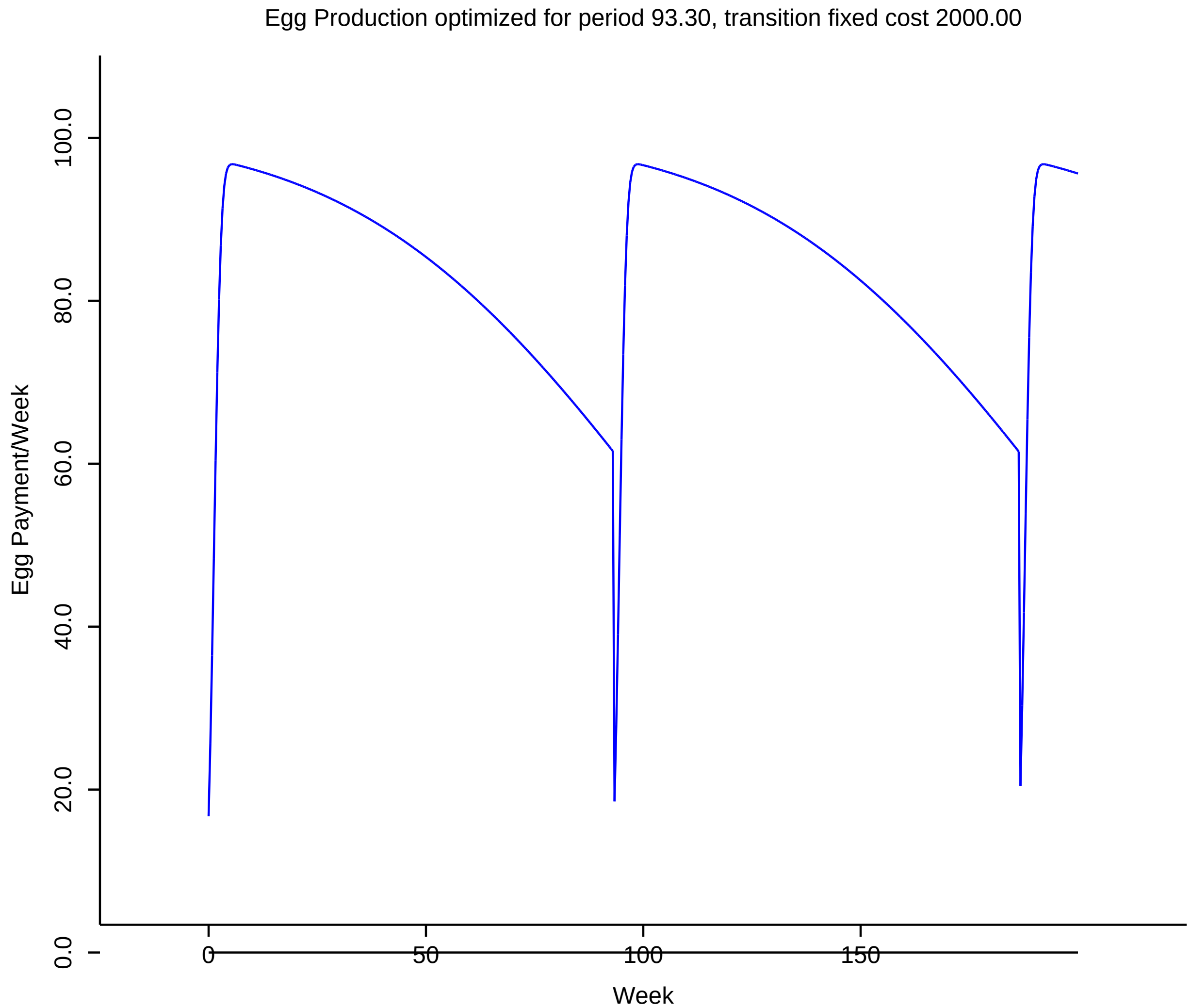

Figure 9: Given the choice of $p(t)$, the weekly payout function, and F, the fixed cost of transition, this is egg payout schedule (ignoring the down-time between flocks).