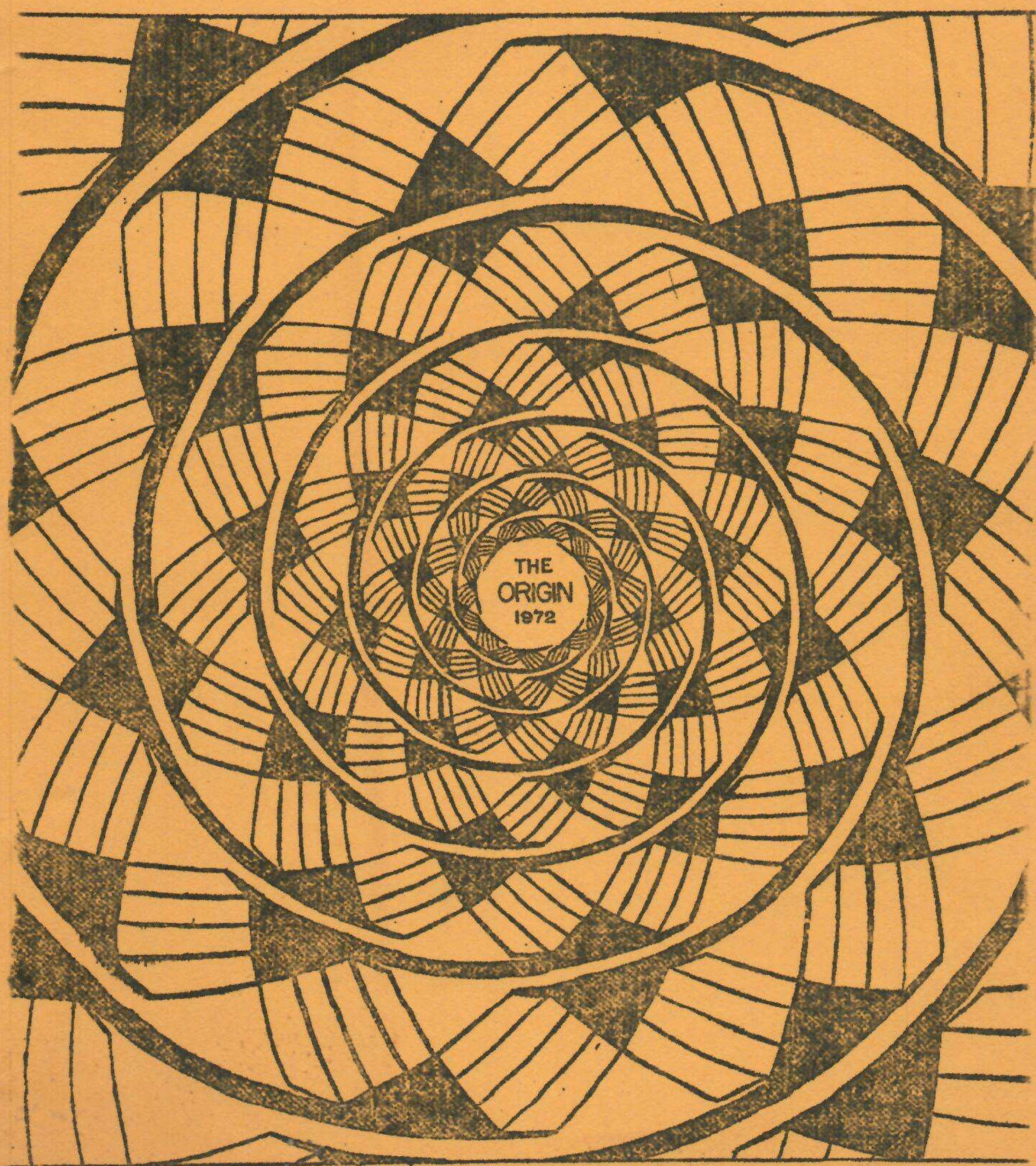
KAPPA MU EPSILON

MATHEMATICS HONOR SOCIETY



BOWLING GREEN STATE UNIVERSITY

This 1971-1972 issue of the Origin
is dedicated to our advisors, Dr.
Waldemar Weber and Dr. Clifford Long
for their assistance and patience.

ABOUT THE COVER

The diagram on the cover simulates a Fraser spiral to focus attention ~~of~~ on The Origin 1972. However, this spiral is only illusory, since it really consists of concentric circles whose radii are determined by the intersections of the logarithmic spirals in the background. These spirals are also equiangular and always intersect one another at right angles, just like the curves that are visible in the seed clusters of daisies. Shirley Oney has provided more details about these remarkable curves in the featured article, which has been condensed from her Honors Program study.

Special recognition should be extended to David Stewart and Janice Csokmay for their work in preparing this issue of The Origin for publication.

Clifford A. Long

Waldemar C. Weber

KME Co-advisors

ABOUT KAPPA MU EPSILON

The minimum requirements for election to membership to the Ohio Alpha Chapter of KAPPA MU EPSILON are a 3.5 average in mathematics, a 2.8 accumulative average, completion of the basic calculus sequence through Math 232, registration in five quarters at Bowling Green State University or its branch campuses, and the certification of the Registrar for eligibility to honoraries in accordance with University policy.

The crest for KAPPA MU EPSILON (back cover) is a shield enclosing the five-pointed star; in the star is the rose $p = \alpha \sin 5\theta$, the symbol of pure mathematics. Around the star are the symbols for the sciences which apply mathematics: at the upper left a book of knowledge, for students and teachers; in the lower left a shamrock and a slide rule, for the engineers; at the upper right a conventionalized butterfly, for the biological sciences; at the lower right a moon and three stars, for the physical sciences; at the bottom the symbol S $\overline{n}|$, for the business world. Above the shield proper is the design of the badge of the fraternity, and below it is a streamer upon which is printed the Greek motto, which means, "Unfold the Glory of Mathematics."

The motto of KAPPA MU EPSILON is "Develop an appreciation for the beauty of mathematics."

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EDITOR'S NOTE

The campus year 1971-1972, while noting such expected changes as the installation of a Ph.D. program in the Mathematics Department, observed a definite change in KAPPA MU EPSILON. Through demand of its new members, the mathematics honorary switched its emphasis from "stimulation in the field of mathematics" to "opportunities in mathematics." One needn't look too far to see why this change was sought; the decaying job market has students everywhere frightened. So with these things in mind, KME began to institute programs and projects which were met with gratifying co-operation and success.

An example of this new emphasis was our second meeting in the fall, appropriately entitled "Is Graduate School For You?" A panel of three "experts" fielded a multiplicity of questions and remarks from an audience of about thirty undergraduates. Dr. Richard Eakin, Assistant Dean of the Graduate School, was able to eloquently respond to the many queries regarding financial assistance. Assistant Professor Thomas Haigh told of his days as a graduate student at the University of Wisconsin and commented on the advantages of "being on both sides of the desk at the same time." The personal aspects of being a grad student and the expectations of the Bowling Green Graduate School were discussed by Tom Reiland. All in all, it was a very enjoyable, informative, and well-attended meeting.

As of this writing, spirit and enthusiasm are high, not only among KME members, but among many non-members as well. Our success can be attributed to many things, but it was the support we received from the people involved that made KME what it was this year.

I personally would like to thank two specific groups. First, for their work on the Origin, I would like to thank Jan for her fanatical dedication to getting this printed on time; the Math secretaries for their technical assistance; and to my "ace reporters", Debbie, Doris, Doyt, and Vickie.

Secondly, I would like to express the gratitude of everyone associated with KME to Dr. Weber and Dr. Long for their hard work, foresight, and, above all, patience.

Dave Stewart
Vice-President
Editor

FROM THE PRESIDENT

This has been an active year for KAPPA MU EPSILON as we have strived to bring about closer relationships between the math majors and faculty members. Our activities this year have centered about our theme "Opportunities in Mathematics".

Early in the year we sponsored a "Meet the Profs" night with over 50 students and 30 of the faculty attending. It was a social hour and an enjoyable time for all - also, a special thanks goes to Barb for making the "punch".

We have followed up our theme by having Karole King, a representative from the Placement Office, give us the 'real' facts of the inert job market. She was very helpful in informing us of the tremendous assistance the Placement Office has to offer. She encouragingly stated that mathematics is an excellent field and one which empolyers consider heavily for a business, government, or teaching job. So maybe there is still hope!

KME sponsors an annual Prize Exam, which is comprised of problems submitted by the faculty. Our 1970-1971 freshman winner was Eric Rosen and our undergraduate winner was Roger Giudici. This year there was no freshman prize given but Doyt Perry and Roger Giudici tied for top honors in the undergraduate exam.

This year 38 new members have been initiated into our honorary. the largest group in our Chapter's history. The initiation ceremony was held at Kaufman's on May 3rd after a delicious buffet dinner. A large number of students and faculty enjoyed the 'new and original' ceremony: new members were escorted by old members, given flowers, and asked to sign the Chapter Roll. The KME Award for Excellence in Teaching was presented to Dr. Herbert Hollister.

Special acknowledgement and thanks go to Dave, whose wit and humor have made this Origin possible and made being president an enjoyable task. Also, our appreciation goes to Dr. Weber and Dr. Long for their encouragement and flexibility in our ideas.

Jan Csokmay

President

WHO'S WHO IN MATHEMATICS AT BOWLING GREEN

1971 - 1972

HASSOON AL-Amiri, Associate Professor. B.S., University of Baghdad; M.A., Ph.D., University of Michigan. Dr. Al-Amiri's main area of study is complex analysis. He advises his students in math to get a broad background in all areas, i.e., algebra, analysis, topology, geometry, etc., and not to grow shallow in areas outside your own specialty. His outside interests include literature and writing plays.

CHARLES H. APPLEBAUM, Assistant Professor. B.S., Case Institute of Technology; M.S.,Ph.D., Rutgers University. Dr. Applebaum is interested in recursive function theory. He chose mathematics because he found it easy and enjoyed it. His 'spare time' is spent listening to music, reading, bowling, or just puttering around the house.

JOSEF BLASS, Assistant Professor. M.A., Warsaw; Ph.D., University of Michigan. Dr. Blass's prime area of research is geometry. He believes mathematics to be a risky field to enter and rewarding to too few. Movies are his favorite method of relaxation.

J. KEVIN BROWN, Assistant Professor. B.A., Notre Dame, Akron University; Ph.D., Ohio State University. Group theory is Dr. Brown's main area. He advises his students to go into mathematics but also to concentrate in other areas for the sake of employment. His outside interests include the stock market, tennis, chess, and oil painting.

RICHARD R. EAKIN, Associate Professor. B.A., Geneva College; M.A., Ph.D., Washington State University. Dr. Eakin's areas of study are probability, mathematical statistics, and combinatorial mathematics. An Assistant Dean of the Graduate School, he believes mathematics continues to provide good employment opportunities relative to other academic fields of study. He enjoys participation in softball, basketball, golf, volleyball, fishing, and ceramics.

RAPHAEL P. FINKELSTEIN, Assistant Professor. B.A., M.A., University of Arizona; Ph.D., Arizona State University. Dr. Finkelstein's research interests are in dioplantine equations, unit algorithms for algebraic number fields, and computer science. In addition to teaching mathematics, he is very interested in linguistics and music.

HUMPHREY SEK-CHING FONG, Assistant Professor. A.B., Carroll College; Ph.D., Ohio State University. Dr. Fong's interests lie in ergodic theory, probability theory, and functional analysis. He believes mathematics to be a very challenging field and that success is dependent upon a genuine interest in solving analytical problems. For recreation, he enjoys paddle ball, ping-pong, bridge, and classical music.

ANDREW M. GLASS, Assistant Professor. B.A., M.A., Cambridge; Ph.D., University of Wisconsin (Madison). Dr. Glass, in his first year here at Bowling Green, is currently interested in partially ordered groups. Since it's customary in England for students to specialize in their field at the age of sixteen, he was greatly influenced by his high school mathematics teacher. His hobbies include reading, croquet, and cricket.

LOUIS C. GRAUE, Professor, Department Chairman. B.S., M.S., University of Chicago; Ph.D., Indiana. Algebra and group theory are Dr. Graue's main areas. He claims that mathematics provides a basic background for all areas and that it is a true test of one's intelligence. Gardening, fishing, and work research on pigeons are his favorite hobbies.

JOHN T. GRESSER, Assistant Professor. B.S., M.S., Ph.D., University o Wisconsin (Milwaukee). Dr. Gresser's specific area of research is cluster set theory. As an undergraduate in Physics, he grew tired of being told to take things for granted, and went into mathematics so he could deduce things for himself. His outside interests include the guitar, skiing, tennis, and transcendental meditation.

J. THOMAS HAIGH, Assistant Professor. B.S., Marquette University; Ph.D., University of Wisconsin. Dr. Haigh's specific area of interest is differential equations. He tells aspirants for graduate school that if they're doers, they'll get through. His outside interests include rugby, modern poetry, and hard rock music.

JOHN L. HAYDEN, Assistant Professor. B.A., University of Missouri; Ph.D., Michigan State. Interested in group theory, Dr. Hayden would like to see more algebra taught at the freshman-sophomore level. He likes mathematics because of its definiteness: it is an exact science, precise yet culturalistic. Formerly in music, he enjoys the piano and playing golf.

THOMAS A. HERN, Assistant Professor. A.B., University of Cincinnati; M.S., Ph.D., Ohio State. Probability is Dr. Hern's main topic of study. He believes mathematics allows one to see the essential parts of a problem without all the camouflaging frills. His hobbies include photography and stereo-hi-fi.

W. CHARLES HOLLAND, Visiting Professor. Ph.D., Tulane University. Algebra, or to be more specific, the structure of ordered groups is Dr. Holland's area of study. Dr. Holland finds it extremely satisfying to work in a field which has concerned scholarly people since the very earliest civilizations. Dr. Holland is an amateur musician and also is interested in literary criticism concerning the Sherlock Holmes stories.

HERBERT A. HOLLISTER, Associate Professor. B.S., Allegheny College; M.A., Ph.D., Univeristy of Michigan. Dr. Hollister's main area of research is partially ordered groups. He has a great respect for mathematicians and their work and believes that students must work diligently at mathematics in order to succeed. For relaxation, he enjoys fishing and golf.

CARLOS S. JOHNSON, Assistant Professor. B.S., Cal Tech; M.A., Ph.D., University of Massachusetts. Dr. Johnson, whose main area of study is lattice theory, is in his second year here at Bowling Green. He chose to go into mathematics because the only thing he enjoyed doing which he could make a living at was math. For relaxation, Dr. Johnson plays the banjo and guitar.

WILLIAM A. KIRBY, Professor. B.A., M.A., University of Wyoming; Ph.D., University of Texas. Dr. Kirby's main area of interest is geometry. He finds mathematics a very exciting discipline and chose to teach because he wanted to make it exciting for others. For recreation, he enjoys bridge, golf, and reading.

DAVID M. KRABILL, Professor. B.A., Wooster College; M.A., Ph.D., Ohio State. Interested in numerical analysis and the use of computers in mathematics, Dr. Krabill recommends a combination of mathematics and computer science - the field is crowded, and students need applied math to complement pure math. His other interests include photography, classical and semi-classical music, and traveling.

J. FREDERICK LEETCH, Professor. B.S., Grove City College; M.A., Ph.D., Ohio State University. Dr. Leetch, whose main area is analysis, believes mathematics is a utilitarian type of tool in that it's used to solve practical problems and helps one to be able to organize in a systematical way. His outside interests include the Naval Reserve Program, sailing, and training his dog. Dr. Leetch has also been active in re-organizing the Problem Solvers Club.

CLIFFORD A LONG, Professor. B.S., M.S., Ph.D., University of Illinois at ~~Chicago~~ Urbana-Champaign. Dr. Long's most recent area of research concerns the geometry of the zeros of polynomials. He believes that mathematics is a good field for those who enjoy the subject, with opportunities for the pure as well as applied mathematician. The 1970 recipient of the KME Teaching Award, Dr. Long's outside interests lie with his family, church, athletics, and woodworking. Dr. Long is the co-advisor of KME.

HOWARD J. MARCUM, Assistant Professor. A.B., A.M., Ph.D., Indiana. Dr. Marcum, whose specfic area of study is algebraic topology, believes the lure of mathematics is that it is just another thing a man can do which offers the ultimate satisfaction of making a personal contribution. He enjoys reading novels and being in the outdoors whenever possible.

HARRY R. MATHIAS, Professor Emeritus. B.A., Indiana Central College; M.A., Indiana University. A general mathematician, Mr. Mathias is in his first year of retirement, but still teaches at the academic centers. He has been the KME Corresponding Secretary for ten years. He enjoys traveling, having been to California, Arizona, and New Orleans since his retirement.

STEPHEN A. McGRATH, Assistant Professor. B.A., Southern Illinois; M.A., Ph.D., Minnesota. Dr. McGrath's main area is ergodic theory. He recommends that undergraduates study applied mathematics - computer science, statistics, operations research, etc. His favorite recreations are chess, baseball, and working on cars.

DAVID B. MERONK, Assistant Professor. B.S., Marquette University; M.S., Ph.D., Notre Dame University. Dr. Meronk is currently studying integral representations of finite groups, arithmetics in algebras, and semigroups. He states that mathematics is a good field because the beauty and consistencies of the various theories are a source of great intellectual satisfaction. He is also interested in reading history and home wine-making.

DEAN A. NEUMANN, Assistant Professor. B.A., Wisconsin State College; M.A., Ph.D., University of Wisconsin. Dr. Neumann's major area of interest is topology. He believes mathematics is a good field for creative and productive people. His outside interests include flying, tropical fish, and music.

VICTOR T. NORTON, Assistant Professor. B.S., Yale (in industrial education), Columbia (math); Ph.D., University of Michigan. Dr. Norton's research is done in differential topology. He went into mathematics because he grew tired of "estimating the cost of large buildings." Although he used to race motorcycles, he now mentions jazz music as one of his favorite interests. He is also to be commended for his organization of photographs of the math department faculty and graduate students, which are displayed outside the mathematics office.

THOMAS V. O'BRIEN, Associate Professor. B.A., M.A., Xavier University; Ph.D., Syracuse. Topology is Dr. O'Brien's main interest. He claims that if you know mathematics well, you will understand the other sciences better. For relaxation, his tastes range from fishing, swimming, and boating to fine dining and good entertainment.

L. DAVID SABBAGH, Assistant Professor. B.S., M.A., Ph.D., Purdue. The 1971 co-recipient of the KME Teaching Award, Dr. Sabbagh's specific area of study is control theory. He became a mathematician because he found the theoretical aspects of engineering more interesting than the laboratory work. His outside interests include sports and music.

MOTUPALLI SATYANARAYANA, Professor. B.A., M.A., Andhrauni, India; Ph.D., University of Wisconsin. Algebra is Dr. Satya's prime area of research. He says his mother wanted him to be a lawyer while he wanted to be an engineer; naturally, he took to mathematics. He enjoys talking world politics, helping children, country music, and doing mathematical research in his rocking chair.

RAYMOND F. SNIPES, Assistant Professor. Ph.D., (chemistry), Yale; Ph.D., (math), University of Virginia. Topological vector spaces are Dr. Snipe's main interest. His work in theoretical chemistry led to his interest in mathematical concepts, and he eventually decided he liked teaching mathematics more than chemistry. He indulges in tennis, paddle ball, and trumpet playing for relaxation.

WALLACE L. TERWILLIGER, Associate Professor. B.S., Clarion State College; M.A., Ph.D., Washington State University. Dr. Terwilliger's specific area of study is general topology. He finds mathematics a very interesting and challenging field, and a basis for many other areas. He is assistant chairman of the department and co-ordinator of schedules. His hobbies are hunting, fishing, bridge, poker, and bull sessions.

RALPH N. TOWNSEND, Associate Professor. B.S., Illinois Wesleyan University; M.S., Ph.D., University of Illinois. Dr. Townsend's main area of interest is real and complex analysis. In addition to being an Assistant Dean in the College of Liberal Arts, he is also interested in ice skating, sailing, astronomy, and music.

WALDEMAR C. WEBER, Assistant Professor. B.S., United States Naval Academy; M.S., Ph.D., University of Illinois. Differential geometry is Dr. Weber's main area of study. Dr. Weber encourages the study of mathematics because it's a great conceptual tool for many areas, and thus has great potential for helping one reach his goals in life. An extremely helpful advisor of KME for 3 years, he studies the Spanish guitar for relaxation.

JAMES G. WILLIAMS, Assistant Professor. B.A., Carlton College; Ph.D., University of California at Berkeley. Dr. Williams' present areas of interest are logic and topology, but is leaning towards mathematics applicable to the social sciences. He believes mathematics has a philosophical value; it is a way of thinking and communicating ideas inexpressible in English. He also is interested in making and refurnishing furniture, and applied sculpture.

DAVID L. WOODRUFF, Assistant Professor. B.A., M.A., Andrews College; Ph.D., Minnesota. Local complex geometry, complex variables, commutative ring theory, and differential topology are Dr. Woodruff's special areas. A mathematician by "historical accident", he enjoys music, collecting glass and antiques, camping, and underwater diving.

THE KME AWARD FOR

EXCELLENCE IN TEACHING MATHEMATICS

The Ohio Alpha Chapter of KAPPA MU EPSILON has established an annual award whose purpose is to stimulate good teaching and improve the already fine quality of instruction in mathematics at Bowling Green. Nominations are made by students, and the recipient is chosen by the Executive Board of KME. All faculty involved in the teaching of mathematics, computer science, or math education are eligible, with the exceptions of the Departmental Chairmen, the KME Faculty Advisors, and the KME Corresponding Secretary. This is a unique award in that it is totally student-controlled and is based solely on the standards of good teaching.

In 1971, the KME Award was given jointly to L. David Sabbagh and V. Frederick Rickey. Students cited their "ability to communicate knowledge through an organized but informal format" and their use of a team-teaching approach to organize and coordinate two large sections of a calculus class. Each instructor retained responsibility for his own section, but students were provided with more opportunities for consultation.

We are proud to announce that the 1972 KME Award has been presented to Herbert A. Hollister. Dr. Hollister was lauded for his "rigorous and meaningful" teaching methods, devotion to his advisees, and availability for personal consultation. He has recently published an Abstract Algebra book which will be used in the 339-403 series. We wish to congratulate Dr. Hollister on his fine achievements here at Bowling Green.

1971 - 1972

KAPPA MU EPSILON

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1972

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Jane Yeagle - junior
Christine Yurga - sophomore

"OPPORTUNITIES IN MATHEMATICS"

Advising Mathematics Majors

Faculty advisors of undergraduate majors in mathematics must be especially careful and honest these days in what they say to prospective graduate students about the employment outlook for a young person who goes on for the Ph.D.

Academic Employment. The United States has gone very quickly from a scarcity of college teachers to a surplus in almost all academic disciplines, and there is no visible factor that could reverse this prospect in the next 18 years. The children who will go to college in that period have already been born and counted. Beyond 18 years we do not yet have the facts but there is no indication that the declining birth rate will sson reverse the trend and bring new growth to college enrollments. College budgets are already being made in the knowledge of these lowered projections so that many faculty vacancies are being cancelled.

Since over 80% of mathematics Ph.D.'s have in the past found employment in college or university teaching, a young person entering graduate work should do so with the severe current and future restriction of academic job opportunities in mind. Competition will necessarily be severe.

Other Employment. Employment of Ph.D. mathematicians in the computer industry, and government laboratories was growing until a few years ago. Now these sectors are depressed. Unlike college teaching, the recession in the computer industry may be temporary, but we cannot now predict when or how much recovery will take place.

An increasing fraction of mathematics students will study mathematics for the reason that they like to do mathematics rather than to qualify for a job. Their Ph.D. degree will give them of certain status and identity, as well as proof of superior ability and achievement. But many of these must seek employment for which mathematics is not a specific qualification. In all walks of life the skills and training of a mathematician should be an advantage to him and to his employer. No other scientific field has such universal applicability in human affairs. But those who go out on their own in this uncertain future must be prepared to assume greater risks than did their earlier counterparts who could confidently look forward to college teaching positions

Committee on Employment
and Educational Policy
of the American
Mathematical Society

THE GOLDEN SECTION

by

Shirley Oney

> "Geometry has two great treasures: one is the theorem of Pythagoras; the other, the division of a line into extreme and mean ratio. The first we may compare to a measure of gold; the second we may name a precious jewel."
>
> Kepler

In Euclid's Elements the following propositions occur:

(1) "To cut a given straight line so that the rectangle contained by the whole and one of the segments is equal to the square of the remaining segment."

(2) "To cut a given finite line in extreme and mean ratio."

The geometric meaning of the golden section is illustrated by the bottom line of figure 1.

DEFINITION. If C divides segment AB such that AB:AC as AC:CB then C is the golden cut or the golden section of segment AB. The ratio AB/AC or AC/CB is called the golden ratio.

The ancient Greeks were familiar with the golden ratio since there is little doubt that it was consciously used by some Greek architects and sculptors, particularly in the structure of the Parthenon. The United States mathematician Mark Barr had this in mind sixty years ago when he gave the ratio the name phi. It is the first letter in the name Phidias, the famous Greek sculptor who used the golden proportion frequently in his work.

To calculate the numerical value of phi, consider figure 1. Let AB have length X. Divide segment AB at C such that AC = 1. Then the length of CB = x - 1. Construct a square on AC to form square ACDE. Then produce ED to F and form rectangle ABFE. The desired proportion is:

$$\frac{X}{1} = \frac{1}{X-1}$$

$$X^2 - X - 1 = 0$$

$$X = \frac{1}{2}(1 \pm \sqrt{5})$$

The positive solution is $X = \frac{1}{2}(1 + \sqrt{5}) = 1.61803398\ldots$ which gives the value of phi to eight decimal places. The positive solution will be denoted by ϕ, and the negative solution will be denoted by $\phi' = \frac{1}{2}(1 - \sqrt{5}) = 0.61803398\ldots$.

It is clear that ϕ' is the negative reciprocal of ϕ, that is, $\phi \cdot \phi' = -1$ since

$$\frac{1}{\phi} = \frac{2}{1+\sqrt{5}} = \frac{\sqrt{5}-1}{2} = -\phi'.$$

Phi is unique in this property: it is the only number that becomes its own reciprocal when subtracted by 1.

THE GOLDEN RECTANGLE

The previous figure is a golden rectangle since the ratio of its sides are in golden proportion. The golden rectangle has many unusual properties. If a square is cut from one end, the remaining figure is a smaller golden rectangle. By cutting off more squares, smaller and smaller golden rectangles will be formed. Successive points marking the division of the sides into golden ratio lie on the logarithmic or equiangular spiral. Its pole is the intersection of the two diagonals shown in figure 2. The sides of the rectangle are almost, but not quite tangential to the curve.

To prove that if a square is cut off from a golden rectangle, then the remaining rectangle is a golden rectangle, consider figure 2 in which ABCD is a golden rectangle. Then $AB:BC = \phi:1$. Let E be the golden cut of AB. Draw EF perpendicular to AB. Then AEFD is a square. The remaining rectangle is a golden rectangle.

Proof: $AB:BC = \phi:1$

Let $AB = \phi$ and $BC = 1$.

Then $EB = \phi - 1 = \frac{\sqrt{5}+1}{2} - 1 = \frac{1}{2}(\sqrt{5} - 1) = \frac{2}{\sqrt{5}+1}$

$BC = 1$

Therefore, $\frac{BC}{EB} = \frac{1}{2/(\sqrt{5}+1)} = \frac{\sqrt{5}+1}{2} = \phi$.

Therefore, EBCF is a golden rectangle.

Continue this process: that is, if EBGH is a square then GHFC is a golden rectangle. We may suppose this process to be repeated indefinitely until the limiting rectangle 0 (indistinguishable from a point) is reached.

PHI AND FIBONACCI

Phi is connected with the Fibonacci sequence which if formed according to the rule that each term is the sum of the two preceeding terms, i.e., $U_{n+1} = U_n + U_{n-1}$. The ratio of successive terms $\frac{U_{n+1}}{U_n}$ approximates more and more closely the value of phi as n increases. The approximations oscillate, being alternately greater than and less than phi. For example,

$$453/280 = 1.6178... < \phi$$
$$733/453 = 1.6181... > \phi \quad .$$

The value of phi was determined from the ratio $\frac{U_{11004}}{U_{11003}}$, each number containing 2300 digits, by the IBM 1401 computer. The digits of U_{n+1}/U_n are identical in every place with U_n/U_{n+1} except that the first begins with 1.6180... and the second begins with 0.6180... . The two ratios were found to coincide to 4,598 decimal places.

Another illustration of the relationship between phi and the Fibonacci series relates to an old geometric fallacy illustrated in figure 3. (It is the same problem proposed by Euclid: "To cut a given straight line so that the rectangle contained by the whole and one of the segments is equal to the square of the remaining segment.")

Construct a square whose side has a length equal to the sum of two consecutive Fibonacci numbers. Divide the square into the sections indicated and fit them together to form a rectangle. Clearly, the areas of the square and the rectangle differ by one square unit. (The explanation of this paradox is that the fit along the diagonal of the rectangle is not exact. There is either an overlap or a gap of one square unit.) Which of the two is larger depends upon the Fibonacci numbers selected. In figure 3, the square is larger than the rectangle by one unit. If instead of 5 and 8 the choice had been 21 and 34, then the rectangle would be larger than the square by one unit. This corresponds to the fact that the consecutive ratios formed from the additive series are alternatively greater than or less than phi.

However, there is one and only one additive series which will produce an exact fit. Since it makes use of the golden section, it has been called the "golden series" which is illustrated below.

$$1, \phi, 1+\phi, 1+2\phi, 2+3\phi, 3+5\phi, 5+8\phi, \ldots \; .$$

Now if we construct a square whose length is equal to the sum of any two consecutive numbers from the golden series, the area of the square will be equal to the area of the rectangle that is formed in the above method. See figure 4.

It should be noted that since $\phi^2 - \phi - 1 = 0$ which implies that $\phi^2 = \phi + 1$, the golden series

$$1, \phi, 1+\phi, 1+2\phi, 2+3\phi, 3+5\phi, \ldots$$

may be written as

$$1, \phi, \phi^2, \phi^3, \phi^4, \phi^5, \phi^6, \ldots .$$

It is the only additive series in which the ratio between any two consecutive terms is constant--the ratio being ϕ.

PENTAGRAM

The pentagram is a rich source of golden ratios since every segment in figure 5 is in golden ratio to the next smallest segment.

In figure 5 above let R be the radius of the circumcircle of pentagon A'B'C'D'E', and let r be the radius of the circumcircle of the pentagon PQRST, and let PQ be a unit length. The following properties as illustrated by Huntley in The Divine Proportion are easily verified.

1. $A'P = \phi$
2. $OA/r = \phi/2$
3. $OA'/r = \phi^2$
4. $OA'/OA = 2\phi$
5. A diagonal such as QS has length ϕ.
6. If X is the point of intersection of two diagonals PR and QS then $\frac{SX}{XQ} = \phi$, $\frac{PX}{XR} = \phi$, and $\frac{B'X}{XT} = \phi$.
7. If SQ produced meets A'B' in V, then, since VQS is parallel to A'D', $\frac{B'V}{VA'} = \frac{B'Q'}{QP} = \frac{B'X}{XT} = \frac{B'S}{SD'} = \phi$
8. The lengths of the six segments B'D', B'S, B'R, RS, RX, XZ are in geometrical progression.

$$B'D' = \phi^3$$
$$B'S = \phi^2$$
$$B'R = \phi$$
$$RS = 1$$
$$RX = \phi^{-1}$$
$$XZ = \phi^{-2}$$

THE GOLDEN TRIANGLE

The golden triangle is an isosceles triangle that has both sides in golden ratio to the base. Each base angle is 72 degrees which is twice the top angle of 36 degrees. This is the golden triangle involved in the construction of the pentagram. If a base angle is bisected, the bisector cuts the opposite side in golden ratio to produce two smaller golden triangles, one of which is similar to the original.

The logarithmic spiral can be constructed from the golden triangle. In figure 6, D is the golden cut of AC, since the bisector of angle B meets AC at D. By this the triangle ABC has been divided into two isosceles triangles which are also golden triangles. The ratio of their areas is $\phi:1$. Bisecting angle C we obtain E, the golden cut of BD, and two more golden triangles are formed. This process, producing a series of gnomons, converges to a limiting point 0, which is the pole of a logarithmic spiral passing successively and in the same order through the three vertices of each of the series of triangles, ...,A,B,C,D,... .

In addition to the constant recurrence of the golden section, a series obeying the Fibonacci rule, $U_{n+1} = U_n + U_{n-1}$, appears in figure 6.

As Huntley illustrates, if HG is a unit length, then the following progression is formed:

$$GF = \phi$$
$$FE = \phi + 1$$
$$ED = 2\phi + 1$$
$$DC = 3\phi + 2$$
$$CB = 5\phi + 3$$
$$BA = 8\phi + 5 \quad .$$

If the other base angles (the ones not already bisected) are joined to the midpoints X,Y,Z,... of the sides opposite them, e.g. CX,DY,EZ,..., then

1. The lengths of these medians form a Fibonacci series, and
2. All the medians pass through the pole 0.

THE EQUIANGULAR SPIRAL

The spiral (figure 7) has various names which correspond to one or another of its features. Descartes discussed it in 1638 and he designated it the equiangular spiral because the angle at which the radius vector cuts the curve at any point is constant. Because its radius increases in geometrical progression, it has been called the geometrical spiral. Jakob Bernoulli named it the logarithmic spiral because of its logarithmic form of the equation. Bernoulli was so fascinated by the mathematical beauty of the curve that he asked that it be engraved on his tombstone with the words--"Though changed I rise unchanged."

The equiangular spiral is truly mathematically beautiful, but its fascination also comes from the fact that it appears so frequently in nature. The fundamental property of the spiral corresponds precisely to the biological principle that governs the growth of the mollusk's shell. The size increases, but the shape remains unaltered. It grows at one end only, each increment of length being balanced by a proportional increase of radius so that its form is unchanged.

To sketch an equiangular spiral (figure 8) first draw a series of lines radiating from a fixed point (the pole) at equal intervals. From a point on one of these lines draw a perpendicular to the next; from the foot of that perpendicular draw another perpendicular to the next line, and so on. A freehand curve may then be drawn through all the points that were found.

THE LOGARITHMIC PROPERTY

The previous method of drawing the spiral determines certain points P,Q,R, but not the intermediate points. The problem of determining intermediate points reduces to that of inserting geometric means between the lengths OP,OQ,... . Therefore, for example, the radius bisecting angle POQ should be of length $\sqrt{(OP)(OQ)}$.

More generally, let POQ be taken as a unit measure of angle, and let OQ/OP = OR/OQ = K. Then if OP = r_o, OQ will be r_oK and OR will be r_oK^2. Then at a point three units of measure from OP, the radius will have length r_oK^3, and so on.

From this we may define all points on the curve by the equation

$$r = r_o K^{\theta}$$

where r is the length of the radius making an angle of θ units with the initial radius OP, of length r_o. This is the polar equation of the curve.

The constant K in this equation may be greater than or less than 1. In the method of drawing suggested, it is less than 1; then the radii OP,OQ,OR,..., diminish as θ increases. However, if K is greater than 1, r increases with θ.

To express the relationship between α and K it is necessary to use calculus. In figure 9 r is increasing as θ increases. So K will be greater than 1. θ will be measured in radians. Draw PM perpendicular to OQ, angle POQ being $\delta\theta$. $OP = r$ and $OQ = r + \delta r$.

Then $MQ = \delta r$ (approximately), and $PM = r\delta\theta$ (approximately). And angle $PQM = \alpha$ (approximately) and in the limit,

$$\cot \alpha = \frac{dr}{rd\theta}.$$

Integrate the above with respect to θ. Then

$$\theta \cot \alpha + \text{constant} = \log_e r.$$

If the initial value of r (i.e., when $\theta = 0$) is r_o, the constant of integration is $\log_e r_o$.

Hence, $\log_e r = \theta \cot \alpha + \log_e r_o$

$$\log_e r - \log_e r_o = \theta \cot \alpha$$

$$\log_e r/r_o = \theta \cot \alpha$$

$$r = r_o e^{\theta \cot\alpha}.$$

(This is the usual form of the polar equation of the equiangular spiral. The curve is also called the logarithmic spiral because of the logarithmic form of the equation.)

THE EQUIANGULAR PROPERTY (FIGURE 10)

Let OU,OV,OW,... be equally spaced radii at any part of the curve, so that the values of θ, namely $\theta_1,\theta_2,\theta_3,\ldots$, are in arithmetic progression. It follows from the equation $r = r_o K^{\theta}$ that the values of $r_1,r_2,r_3,\ldots$ are in geometrical progression, for

$$r_2 = r_1 K^{\theta_2-\theta_1} \quad \text{and} \quad r_3 = r_2 K^{\theta_3-\theta_2}.$$

Therefore, $r_2/r_1 = K^{\theta_2-\theta_1} = K^{\theta_3-\theta_2} = r_3/r_2$.

Hence the triangles OUV,OVW,..., are similar. If U,V,W,..., are taken very close together, the angles OUV,OVW,..., will be very nearly the angles between the tangents to the curve and the radii OU,OV,... . But angles OUV,OVW,..., are all equal. Hence in the limit, the angle between the tangent to the curve and the radius is constant.

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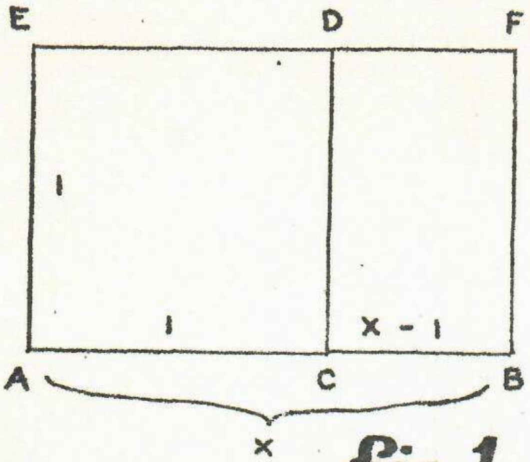
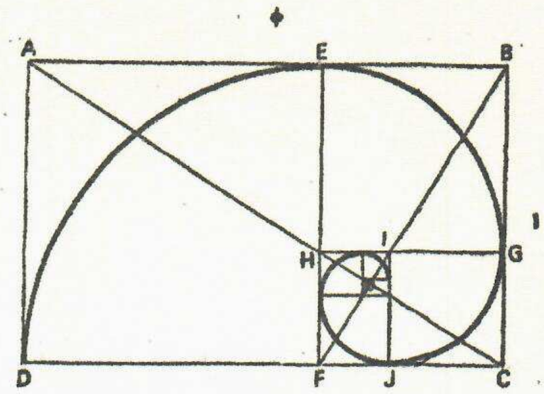


fig 1



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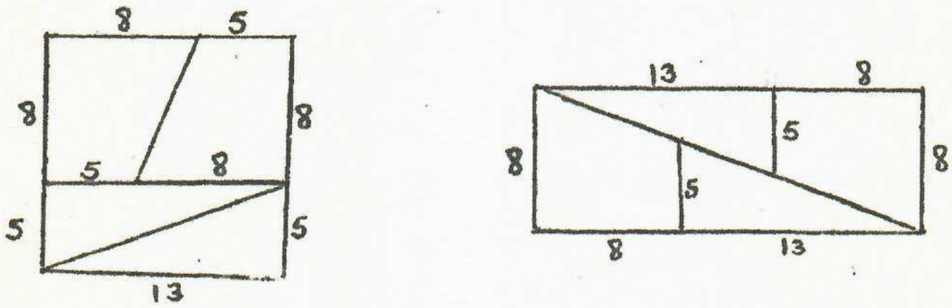
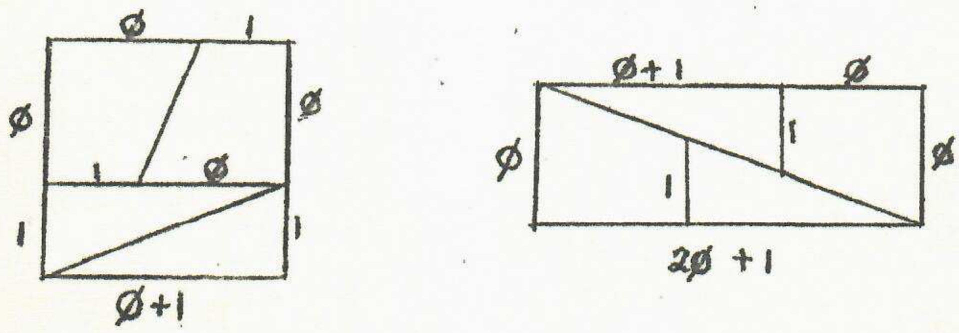


$\text{Area} = 13^2 = 169$

$\text{Area} = 8 \times 21 = 168$

fig 3



$$\begin{aligned}\text{Area} &= (\phi + 1)^2 \\ &= \phi^2 + 2\phi + 1 \\ &= (\phi+1) + 2\phi + 1 \\ &= 3\phi + 2\end{aligned}$$

(Recall: $\phi^2 - \phi - 1 = 0$)

$$\begin{aligned}\text{Area} &= (2\phi + 1)\phi \\ &= 2\phi^2 + \phi \\ &= 2(\phi + 1) + \phi \\ &= 3\phi + 2\end{aligned}$$

fig 4

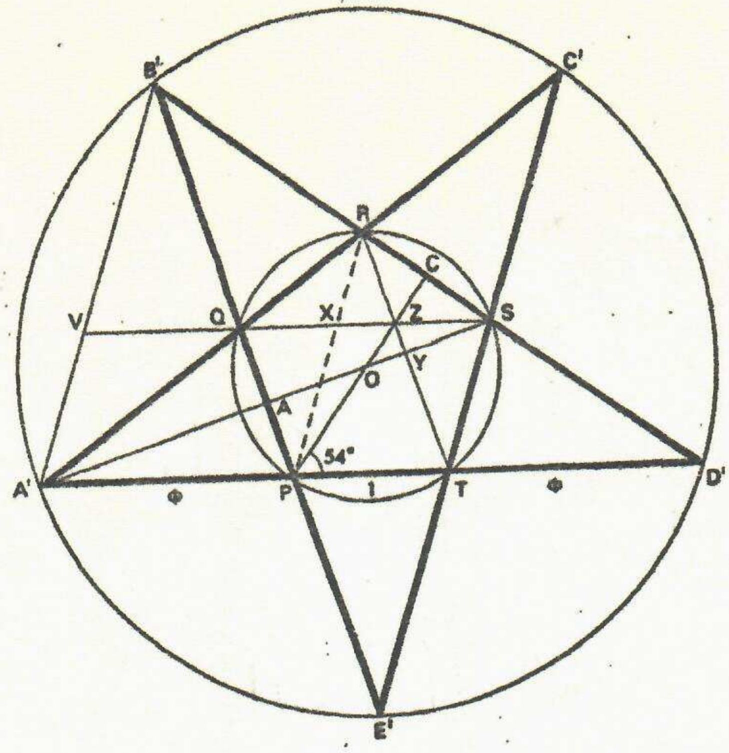


fig 5

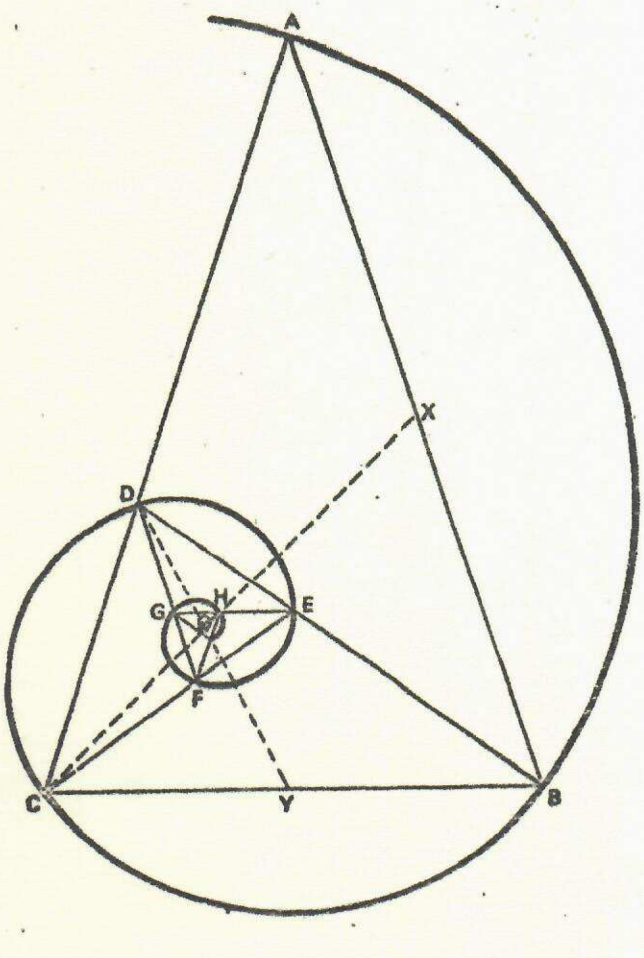


fig 6

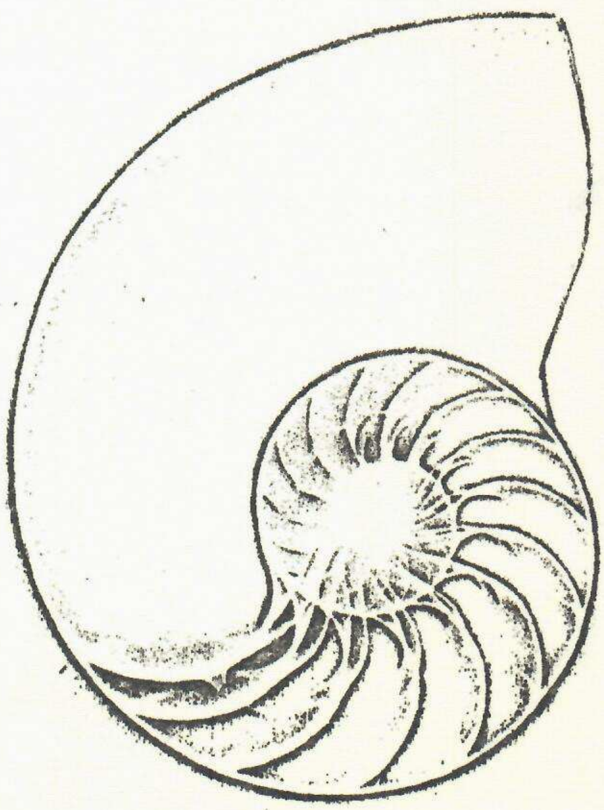


fig 7

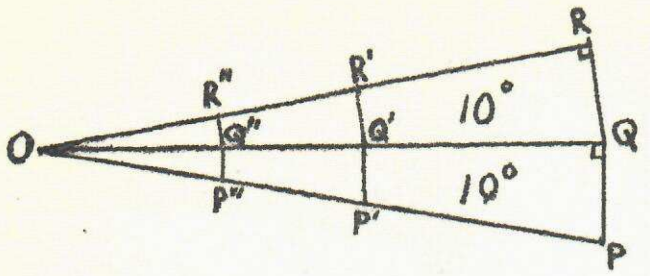


fig 8

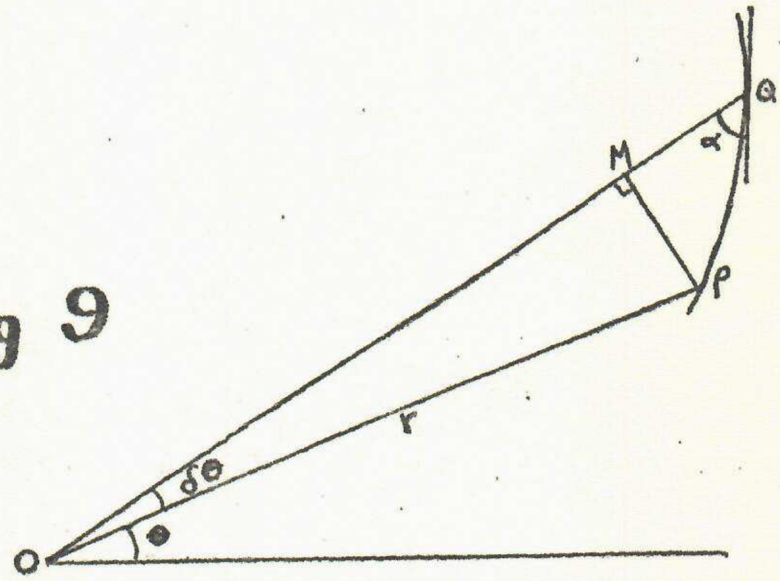


fig 9

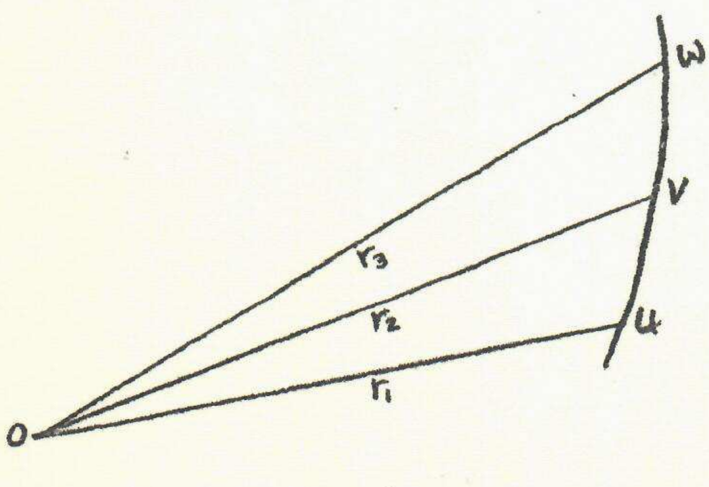
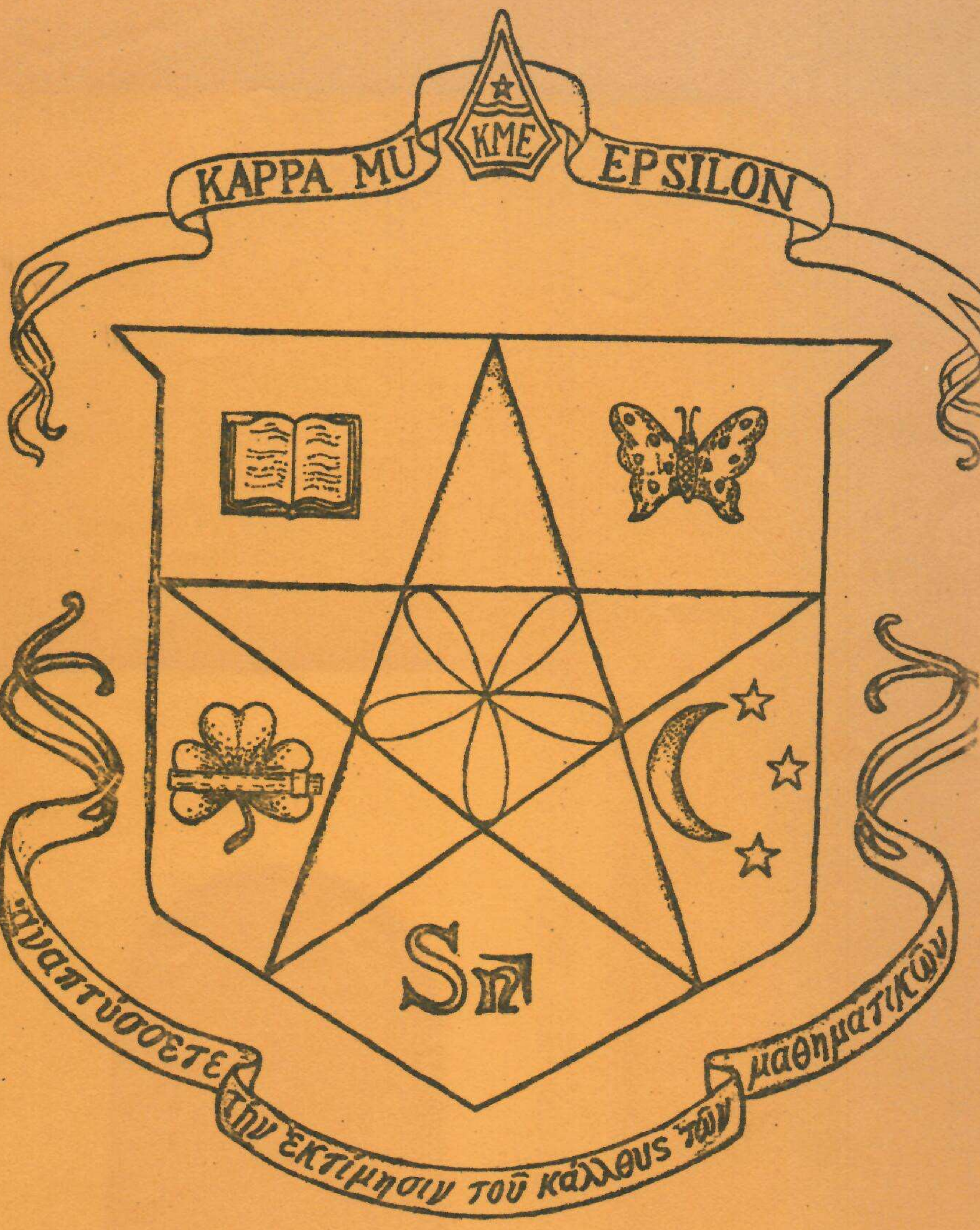


fig 10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