$$\underbrace{N_{n+1}}(x) = N_n(x) + q(x) \leftarrow n+1^{\text{st}} \text{ degree}$$

stay out of the way of $N_n(x)$ for $x_0, \dots, x_n$

And take care of x_{n+1}

$$\left(N_{n+1}(x_{n+1}) = f(x_{n+1}) \right)$$

$$q(x_i) = \begin{cases} 0 & i \in \{0, \dots, n\} \\ f(x_i) - N_n(x_i) & i = n+1 \end{cases}$$

$$q(x) = \frac{(x-x_0)}{(x_{n+1}-x_0)} \frac{(x-x_1)}{(x_{n+1}-x_1)} \cdots \frac{(x-x_n)}{(x_{n+1}-x_n)} \cdot \left(f(x_{n+1}) - N_n(x_{n+1}) \right)$$

$$N_{n+1}(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + \cdots + q_{n+1}(x)$$

a_0: $f(x_0)$

a_1: $\dfrac{f(x_1) - f(x_0)}{x_1 - x_0} = \dfrac{f(x_{n+1}) - N_0(x_1)}{x_1 - x_0}$

a_2: $f(x_2) - N_1$

x_0	$f(x_0)$	$\dfrac{f(x_1)-f(x_0)}{x_1-x_0}$
x_1	$f(x_1)$	$\dfrac{f(x_2)-f(x_1)}{x_2-x_1}$
x_2	$f(x_2)$	

a_2

$$a_2 = \frac{f(x_2) - N_1(x_2)}{(x_2-x_0)(x_2-x_1)}$$

$$= \frac{f(x_2) - \left[f(x_0) + \frac{f(x_1)-f(x_0)}{x_1-x_0}(x_2-x_0) \right]}{(x_2-x_0)(x_2-x_1)}$$

$$= \frac{f(x_2) - \left[f(x_0) + \frac{f(x_1)-f(x_0)}{x_1-x_0}(x_2-x_1+x_1-x_0) \right]}{(x_2-x_0)(x_2-x_1)}$$

$$= \frac{f(x_2) - \left[\cancel{f(x_0)} + \frac{f(x_1)-f(x_0)}{x_1-x_0}(x_2-x_1) + f(x_1) - \cancel{f(x_0)} \right]}{(x_2-x_0)(x_2-x_1)}$$

$$= \frac{f(x_2) - f(x_1) - \frac{f(x_1)-f(x_0)}{x_1-x_0}(x_2-x_1)}{(x_2-x_0)(x_2-x_1)}$$

$$= \frac{\frac{f(x_2)-f(x_1)}{x_2-x_1} - \frac{f(x_1)-f(x_0)}{x_1-x_0}}{(x_2-x_0)}$$

Hmm.... looks kind of like a 2nd derivative....