Homework 2.1

Andy Long, Spring, 2024

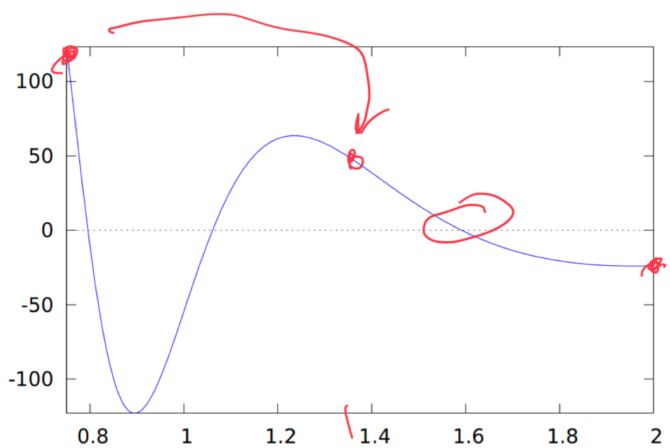
Section 2.1, pp. 47--, #1, 2a-e, 4a-e, 7, 9, 13, 17, 26

The first problems (through 9) are solved in Octave:

https://octav.onl/teaTime2_1

The rest of the problems (13, 17, 26) are here:

13. The graph of $f(x)$ over the interval $[0.75, 2]$ is shown below. Notice $f(x)$ has three roots on this interval: approximately .795, 1.06, and 1.59. To which of the three roots does the bisection method converge if we let $a = .75$ and $b = 2$? How do you know?



The third: the signs of m and b are different, so that interval will be chosen.

17. Suppose you are using the bisection method on an interval of length 3. How many iterations are necessary to guarantee accuracy of the approximation to within 10^{-6}?

At each iteration, the interval is cut in half: 3/2^n<=10^(-6). We can solve it, and then take the ceiling:

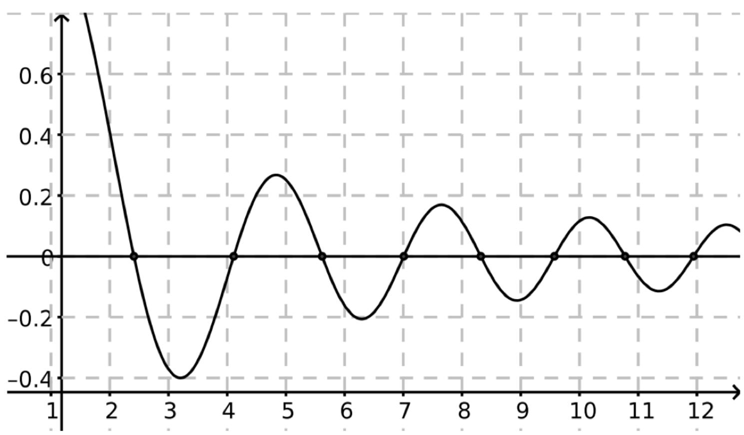
2^n=3000000, so Ceiling[Log[3000000]/Log[2]]

n = 22

In[∘]:=
```
Ceiling[Log[3 000 000] / Log[2]]
```

Out[∘]= 22

26. The function shown has roots at approximately 2.41, 4.11, 5.62, 7.01, 8.32, 9.57, 10.78, and 11.94. To which root will the bisection method converge with the given starting interval?



a. [2,3] - 2.41 } only roots in the interval

b. [6,8] - 7.01

c. [2,6] → [2,4] → 2.41

d. [5,9] → [5,7] → 5.62

e. [10,12] → [10,11] because the 1st comparison is w/ the left endpoint

∴ 10.78

(a) $[2, 3]$

(b) $[6, 8]$

(c) $[2, 6]$

(d) $[5, 9]$

(e) $[10, 12]$ Note: the assumptions of the bisection are not met on this interval. Nonetheless, the method as outlined in the pseudo-code *will* converge to a root!