Chapter 1, Section 1: Homework:
pp. 8-9, #3-5, 8, 11, 18, 20

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3. Calculate the absolute error in approximating p by $\tilde{p}$.

(a) $p = 123; \quad \tilde{p} = \frac{1106}{9}$ [S]

(b) $p = \frac{1}{e}; \quad \tilde{p} = .3666$

3. (c) $p = 2^{10}; \quad \tilde{p} = 1000$ [S]

(d) $p = 24; \quad \tilde{p} = 48$

(e) $p = \pi^{-7}; \quad \tilde{p} = 10^{-4}$ [S]

(f) $p = (0.062847)(0.069234); \quad \tilde{p} = 0.0042$

In[525]:=
```
absError[p_, ptilde_] := N[Abs[p - ptilde]]
```

In[526]:=
```
absError[123, 1106 / 9]
absError[1 / E, .3666]
absError[2 ^ 10, 1000]
absError[24, 48]
absError[Pi ^ (-7), 10 ^ (-4)]
absError[(0.062847) (0.069234), 0.0042]
```

Out[526]= 0.111111

Out[527]= 0.00127944

Out[528]= 24.

Out[529]= 24.

Out[530]= 0.000231094

Out[531]= 0.000151149

---

4. Calculate the relative errors in the approximations of question 3.

In[532]:=
```
relError[p_, ptilde_] := N[Abs[p - ptilde] / Abs[p]]
```

In[533]:=
```
relError[123, 1106 / 9]
relError[1 / E, .3666]
relError[2^10, 1000]
relError[24, 48]
relError[Pi^(-7), 10^(-4)]
relError[(0.062847) (0.069234), 0.0042]
```

Out[533]= 0.000903342

Out[534]= 0.00347788

Out[535]= 0.0234375

Out[536]= 1.

Out[537]= 0.697971

Out[538]= 0.0347378

5. How many significant digits of accuracy do the approximations of question 3 have?

In[539]:=
```
sigDigits[p_, ptilde_] := N[-Log10[relError[p, ptilde]]]
```

In[540]:=
```
sigDigits[123, 1106 / 9]
sigDigits[1 / E, .3666]
sigDigits[2^10, 1000]
sigDigits[24, 48]
sigDigits[Pi^(-7), 10^(-4)]
sigDigits[(0.062847) (0.069234), 0.0042]
```

Out[540]= 3.04415

Out[541]= 2.45869

Out[542]= 1.63009

Out[543]= 0.

Out[544]= 0.156163

Out[545]= 1.4592

8. The number in question 7 is an approximation of $1/\pi$. Using Octave, find the absolute and relative errors in the approximation.

In[546]:=
```
absError[1103 Sqrt[8] / 9801, 1 / Pi]
relError[1103 Sqrt[8] / 9801, 1 / Pi]
sigDigits[1103 Sqrt[8] / 9801, 1 / Pi]

N[1103 Sqrt[8] / 9801, 20]
N[1 / Pi, 20]
```

Out[546]= 7.74332×10^{-9}

Out[547]= 2.43264×10^{-8}

Out[548]= 7.61392

Out[549]= 0.31830987844047012322

Out[550]= 0.31830988618379067154

11.

11. All of these equations are mathematically true. Nonetheless, floating point error causes some of them to be false according to Octave. Which ones? HINT: Use the boolean operator **==** to check. For example, to check if $\sin(0) = 0$, type **sin(0)==0** into Octave. **ans=1** means true (the two sides are equal according to Octave—no round-off error) and **ans=0** means false (the two sides are not equal according to Octave—round-off error).

(a) $(2)(12) = 9^2 - 4(9) - 21$

(b) $e^{3\ln(2)} = 8$

(c) $\ln(10) = \ln(5) + \ln(2)$

(d) $g(\frac{1+\sqrt{5}}{2}) = \frac{1+\sqrt{5}}{2}$ where $g(x) = \sqrt[3]{x^2 + x}$

(e) $\lfloor 153465/3 \rfloor = 153465/3$

(f) $3\pi^3 + 7\pi^2 - 2\pi + 8 = ((3\pi + 7)\pi - 2)\pi + 8$

In[551]:=
```
2 * 12 == 9^2 - 4 * 9 - 21
E^(3 Log[2]) == 8
Log[10] == Log[5] + Log[2]

g[x_] := CubeRoot[x^2 + x];
g[(1 + Sqrt[5]) / 2] == (1 + Sqrt[5]) / 2
EqualTo[g[(1 + Sqrt[5]) / 2]][(1 + Sqrt[5]) / 2]

Floor[153 465 / 3] == 153 465 / 3
3 Pi^3 + 7 Pi^2 - 2 Pi + 8 == ((3 Pi + 7) Pi - 2) Pi + 8
```

Out[551]= True

Out[552]= True

Out[553]= True

Out[555]= $\left(\frac{1}{2}\left(1+\sqrt{5}\right)+\frac{1}{4}\left(1+\sqrt{5}\right)^2\right)^{1/3} == \frac{1}{2}\left(1+\sqrt{5}\right)$

Out[556]= $\frac{1}{2}\left(1+\sqrt{5}\right) == \left(\frac{1}{2}\left(1+\sqrt{5}\right)+\frac{1}{4}\left(1+\sqrt{5}\right)^2\right)^{1/3}$

Out[557]= True

Out[558]= True

```
2*12 == 9^2 - 4*9 - 21
e^(3*log(2)) == 8
log(10) == log(5) + log(2)

g = @(x) (x^2 + x)^(1/3)
g((1 + sqrt(5))/2) == (1 + sqrt(5))/2

floor(153465/3) == 153465/3
3*pi^3 + 7*pi^2 - 2*pi + 8 == ((3*pi + 7)*pi - 2)*pi + 8
```

```
octave:4> source("hw1_1.m")
ans = 1
ans = 0
ans = 0
g =

@(x) (x ^ 2 + x) ^ (1 / 3)

ans = 1
ans = 1
ans = 0
```

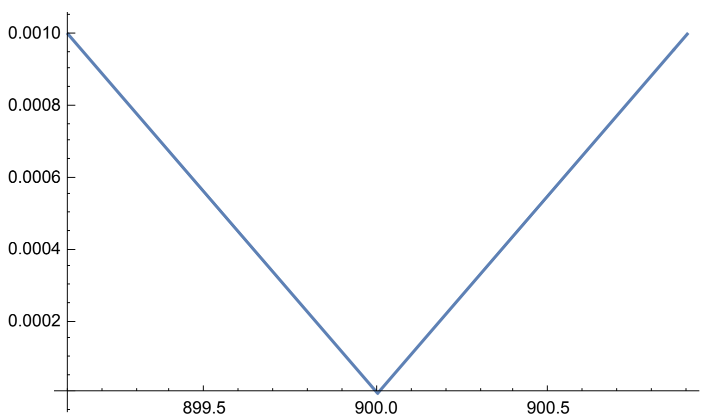
18.

18. Suppose $\tilde{p}$ must approximate p with relative error at most 10^{-3}. Find the largest interval in which $\tilde{p}$ must lie if $p = 900$.

```
In[559]:= {p0, p1} = pstar /. Solve[relError[900, pstar] == 10^(-3), pstar]
Plot[relError[900, pstar], {pstar, p0, p1}]
```

Out[559]= {899.1, 900.9}

Out[560]=



20.

20. The golden ratio, $\frac{1+\sqrt{5}}{2}$, is found in nature and in mathematics in a variety of places. For example, if F_n is the n^{th} Fibonacci number, then

$$\lim_{n\to\infty} \frac{F_{n+1}}{F_n} = \frac{1+\sqrt{5}}{2}$$

Therefore, F_{11}/F_{10} may be used as an approximation of the golden ratio. Find the relative error in this approximation. HINT: The Fibonacci sequence is defined by $F_0 = 1$, $F_1 = 1$, $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$.

Note: Mathematica (and a lot of others, including I) define Fibonacci[0] as 0 (we're offset from this sequence by one place).

```
In[561]:= Fibonacci[0]
f11 = Fibonacci[12]
f10 = Fibonacci[11]
NumberForm[N[GoldenRatio, 30], 30]
relError[f11 / f10, GoldenRatio]
NumberForm[N[Fibonacci[110] / Fibonacci[109], 30], 30]

Out[561]= 0

Out[562]= 144

Out[563]= 89

Out[564]//NumberForm=
1.61803398874989484820458683437

Out[565]= 0.0000348958

Out[566]//NumberForm=
1.61803398874989484820458683437
```