MAT225 Section Summary: 5.3

Diagonalization
Summary

diagonalizable: A square matrix A is diagonalizable if A is similar to a diagonal matrix. That is, if $A = PDP^{-1}$ for some diagonal matrix D.

The Diagonalization Theorem: $A_{n \text{ x } n}$ is diagonalizable if and only if A has n linearly independent eigenvectors. Moreover, $A = PDP^{-1}$ (where D is diagonal) if and only if the columns of P are n linearly independent eigenvectors of A. In this case, the diagonal entries of D are the eigenvalues.

Example: #2, p. 325

Rewrite the equation $A = PDP^{-1}$ in the form $AP = PD$ to understand what is going on! This is just the eigenvalue equation in partitioned form:

$$A[\mathbf{v}_1 \ \mathbf{v}_2 \ \ldots \ \mathbf{v}_n] = [\lambda_1\mathbf{v}_1 \ \lambda_2\mathbf{v}_2 \ \ldots \ \lambda_n\mathbf{v}_n]$$

Theorem 6: An n x n matrix with n distinct eigenvalues is diagonalizable.

Example: #10, p. 326

Theorem 7: Let A be an n x n matrix whose distinct eigenvalues are $\lambda_1, \lambda_2, \ldots, \lambda_p$.

1. For $1 \leq k \leq p$, the dimension of the eigenspace for λ_k is less than or equal to the multiplicity of the eigenvalue λ_k.
2. The matrix A is diagonalizable if and only if the sum of the dimensions of the distinct eigenspaces equals n.
3. If A is diagonalizable, and B_k is a basis for the eigenspace corresponding to λ_k, then the collection of the bases $B_1, \ldots, B_p$ forms an eigenvector basis for $\mathbb{R}^n$.

Example: #33, p. 326