MAT225 Section Summary: 1.3
Vector Equations

1. Definitions

- **vector**
  a matrix with only a single column ("column vector"). The entries are called the **components** of the vector.
- **zero vector**
  the vector whose components are all 0.
- **one vector**
  the vector whose components are all 1.
- **scalar multiple** of a vector
  a product of a constant ("**scalar**") and a vector, the operation being carried out component-wise.
- **vector sum**: the vector created by adding two vectors, the sums being carried out component-wise. The sum of vectors can be found using the "parallelogram rule": the butt of vector $\mathbf{v}_2$ is placed at the tip of the vector $\mathbf{v}_1$, and the vector from the butt of $\mathbf{v}_1$ to the tip of $\mathbf{v}_2$ is the sum.
- **linear combination** of vectors
  any sum of vectors scaled by coefficients.

  Example: #4, p. 37

- **span**
  the span of a set of vectors $\{\mathbf{v}_1, ...\mathbf{v}_p\}$ is the subspace generated by linear combinations of the vectors $\mathbf{v}_i$. The span represents the set of vectors that can be solutions of the system

$$a_1\mathbf{v}_1 + \ldots + a_p\mathbf{v}_p = \mathbf{b}$$

Q: What is the geometry of a span? What cases should be considered?

2. Properties/Tricks/Hints/Etc.

- The vector equation

  $$a_1\mathbf{v}_1 + \ldots + a_p\mathbf{v}_p = \mathbf{b}$$

  has the same solution as the linear system whose augmented matrix is

  $$[\mathbf{a_1} \quad \mathbf{a_2} \quad \ldots \quad \mathbf{a_p} \quad \mathbf{b}]$$

  Example: ¡form method=POST action=http://sappho.nku.edu/tildaroonielonga/cgi-bin/Octave/octave.cgi¿ ¡INPUT NAME=override TYPE=hidden VALUE=demofile¿ ¡INPUT NAME=example TYPE=hidden VALUE=mat225day05.9.r ¡INPUT NAME=color TYPE=hidden VALUE=on¿ ¡input TYPE=SUBMIT VALUE='#9, p. 37'¿ ¡/form¿

  Example: ¡form method=POST action=http://sappho.nku.edu/tildaroonielonga/cgi-bin/Octave/octave.cgi¿ ¡INPUT NAME=override TYPE=hidden VALUE=demofile¿ ¡INPUT NAME=example TYPE=hidden VALUE=mat225day05.12 ¡INPUT NAME=color TYPE=hidden VALUE=on¿ ¡input TYPE=SUBMIT VALUE='#12, p. 38'¿ ¡/form¿

- Two vectors are equal only if they have the same dimensions, and their components are the same.

- Algebraic properties of the **vector space** $\mathbb{R}^n$: for all $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ in $\mathbb{R}^n$ and all scalars c and d,

  (a) $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$

  (b) $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$

  (c) $\mathbf{u} + \mathbf{0} = \mathbf{u}$

  (d) $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$

  (e) $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$

  (f) $(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$

(g) $c(d\mathbf{u}) = (cd)\mathbf{u}$

(h) $1\mathbf{u} = \mathbf{u}$

3. Summary

Vectors provide a wonderful way for us to write systems of equations compactly. You should already be familiar with two-d and three-d vectors from calculus classes. We now want to extend notions from those spaces into n-dimensional space. For example, vector addition is carried out component-wise.

The interesting new concept introduced in this section is that of **span**: roughly, the span of a set of vectors $\{\mathbf{v}_1, ...\mathbf{v}_p\}$ is the subspace generated by linear combinations of the vectors $\mathbf{v}_i$. The span represents the set of vectors that can be solutions of the system

$$a_1\mathbf{v}_1 + \ldots + a_p\mathbf{v}_p = \mathbf{b}$$

Example: #21, p. 38

Example: #27, p. 38