You can also view this case study in the following formats:


## Diet Problems

## Text Reference: Section 1.10, p. 93

The purpose of this set of exercises is to provide examples of vector equations which result from balancing nutrients in a diet.

Section 1.10 shows how use a vector equation

$$
x_{1} \mathbf{a}_{1}+x_{2} \mathbf{a}_{2}+\ldots+x_{n} \mathbf{a}_{n}=\mathbf{b}
$$

to model a diet with a specified nutritional intake. Each vector $\mathbf{a}_{i}$ lists the nutrient composition of one unit (usually 100 grams) of foodstuff, and the corresponding weight $x_{i}$ is the variable that represents the amount (number of units) of that foodstuff to be used in the diet. The vector $\mathbf{b}$ lists the amount of each nutrient that must be in the diet.

Table 2 below is a listing of the nutritional value of many foods found in a typical kitchen. The nutrients are given per 100 grams of foodstuff. This data is taken from the USDA Nutrient Database for Standard Reference available at the United States Department of Agriculture website (http://www.nal.usda.gov/fnic/foodcomp). The columns represent respectively the following foodstuffs: beef, brussels sprouts, carrots, chicken soup, egg, feta cheese, grapefruit, lentils, lettuce, milk, mushrooms, oil, onion, rice, salad dressing, salmon, soy sauce, spinach, tomato, and vanilla ice cream. Table 1 gives the standard serving size for each of these foodstuffs, and also gives a key to the columns in Table 2.

## Questions:

1. Low carbohydrate diets are popular for weight loss. Compute (by hand) the amount of carbohydrates in each of the following dishes, and determine which would be better for such a dieter to choose. You will first need to use Table 1 to convert the kitchen measures into 100 gram units, then use Table 2 to find the amount of carbohydrates in each ingredient.

Spinach omelet: $1 / 4$ cup spinach, 2 eggs, $1 / 8$ cup milk, $1 / 2$ Tbsp. oil
Greek salad: 1 cup lettuce, $1 / 4$ cup feta cheese, $1 / 2$ of a tomato, $1 / 8$ cup salad dressing
2. To make a stir fry, fry beef and onions in a wok with oil, and top it with soy sauce.
a) You would like to make a stir fry with a total of 6 g carbohydrates, 50 g protein, and 3.5 mg vitamin C , and you'd like this dish to contain only 575 calories. Use Table 2 to set up a matrix equation which could be used to determine whether it is possible to make such a stir fry. Describe the steps you take to produce the vectors in the equation.
b) Find a precise recipe for the stir fry in part a). Convert your amounts to common kitchen measures using Table 1.
3. Table 2 has been incorporated into the matrix $A$ which accompanies this exercise set. What does the $j^{\text {th }}$ column in this matrix represent? Which entry in this matrix tells you how much vitamin C is found in 100 g of vanilla ice cream?
4. A particularly math-savvy sumo wrestler wanted to adhere to a strict diet to maintain his weight and strength. Table 3 lists his desired nutritional intake for one day. The entries in Table 3 are stored in the vector $\mathbf{v}_{1}$ which accompanies this exercise set. Using Table 2 he was able to decide on an optimal diet to give him this combination of nutrients. How much of each of the above foods were in his diet?
5. The United States Food and Drug Administration (FDA) provides Recommended Daily Values for use on food labels. Table 4 gives the FDA's recommendations, which are also stored in the vector $\mathrm{v}_{2}$ which accompanies this exercsise set. Is it possible to combine the foods from the table to approximate these nutritional values?

| Key Number | Foodstuff | Serving Size |
| ---: | :--- | :---: |
| 1 | Beef | 6 oz=170g |
| 2 | Brussels Sprouts | $1 / 2$ cup $=78 \mathrm{~g}$ |
| 3 | Carrots | 1 carrot=61g |
| 4 | Chicken Soup | 1 cup $=240 \mathrm{~g}$ |
| 5 | Egg | 1 egg $=61 \mathrm{~g}$ |
| 6 | Feta Cheese | $1 / 4$ cup $=38 \mathrm{~g}$ |
| 7 | Grapefruit | $1 / 2$ fruit $=123 \mathrm{~g}$ |
| 8 | Lentils | 1 cup $=198 \mathrm{~g}$ |
| 9 | Lettuce | $1 / 2$ cup $=28 \mathrm{~g}$ |
| 10 | Milk | 1 cup $=244 \mathrm{~g}$ |
| 11 | Mushrooms | $1 / 2$ cup $=35 \mathrm{~g}$ |
| 12 | Oil | 1 Tbsp=13.5g |
| 13 | Onion | 1 onion=110g |
| 14 | Rice | 1 cup $=158 \mathrm{~g}$ |
| 15 | Salad Dressing | 1 cup=250g |
| 16 | Salmon | $1 / 2$ fillet $=124 \mathrm{~g}$ |
| 17 | Soy Sauce | 1 Tbsp=18g |
| 18 | Spinach | 1 cup $=180 \mathrm{~g}$ |
| 19 | Tomato | 1 tomato $=123 \mathrm{~g}$ |
| 20 | Vanilla Ice Cream | $1 / 2$ cup=66g |

Table 1: Serving Sizes of Various Foodstuffs

| Nutrient (units) | Foodstuff (see key for names) |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Calories (kcal) | 215 | 39 | 43 | 73 | 152 | 263 | 30 | 116 | 14 | 61.44 |
| Protein (g) | 26 | 2.55 | 1.03 | 5.3 | 10.33 | 14.2 | . 55 | 9.02 | 1.62 | 3.29 |
| Fat (g) | 11.5 | . 51 | . 19 | 2.5 | 11.44 | 21.3 | . 1 | . 38 | . 2 | 3.34 |
| Carbohydrates (g) | 0 | 8.6 | 10.1 | 7.1 | 1.04 | 4.09 | 7.68 | 20.14 | 2.37 | 4.66 |
| Calcium (mg) | 7 | 36 | 27 | 10 | 42 | 492.5 | 11 | 19 | 36 | 119.4 |
| Iron (mg) | 3.1 | 1.2 | . 5 | . 6 | 1.19 | . 65 | . 12 | 3.33 | 1.1 | . 05 |
| Magnesium (mg) | 27 | 20 | 15 | 4 | 9 | 19.2 | 8 | 36 | 6 | 13.44 |
| Phosphorus (mg) | 211 | 56 | 44 | 30 | 148 | 337 | 9 | 180 | 45 | 93.4 |
| Potassium (mg) | 367 | 317 | 323 | 45 | 101 | 61.8 | 129 | 369 | 290 | 151.5 |
| Sodium (mg) | 69 | 21 | 35 | 354 | 270 | 1116 | 0 | 2 | 8 | 49 |
| Zinc (mg) | 5290 | . 33 | . 2 | . 4 | . 92 | 2.88 | . 07 | 1.27 | . 25 | . 38 |
| Copper (mcg) | . 143 | . 083 | . 047 | . 1 | . 013 | . 032 | . 044 | . 251 | . 037 | . 01 |
| Vitamin C (mg) | 0 | 62 | 9.3 | 0 | 0 | 0 | 38.1 | 1.5 | 24 | . 94 |
| Thiamine (mg) | . 11 | . 107 | . 097 | . 03 | . 044 | . 154 | . 034 | . 169 | . 1 | . 038 |
| Riboflavin (mg) | . 25 | . 08 | . 059 | . 07 | . 399 | . 844 | . 02 | . 073 | . 1 | . 162 |
| Niacin (mg) | 4.63 | . 607 | . 928 | 1.8 | . 058 | . 991 | . 191 | 1.06 | . 5 | . 084 |
| Pantothenic Acid (mg) | . 34 | . 252 | . 197 | 15 | . 934 | . 967 | . 283 | . 638 | . 17 | . 314 |
| Vitamin B6 (mg) | . 4 | . 178 | 147 | . 02 | . 109 | . 424 | . 042 | . 178 | . 047 | . 042 |
| Vitamin B12 (mcg) | 2.27 | 0 | 0 | . 13 | . 7 | 1.69 | 0 | 0 | 0 | . 357 |
| Vitamin A (IU) | 0 | 719 | 28129 | 509 | 654 | 447 | 259 | 8 | 2600 | 126 |


| Nutrient (units) | Foodstuff (see key for names) |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| Calories (kcal) | 25 | 884 | 38 | 130 | 448.8 | 149 | 60 | 23 | 21 | 201 |
| Protein (g) | 2.09 | 0 | 1.16 | 2.69 | 0 | 25.56 | 10.51 | 2.9 | . 85 | 3.5 |
| Fat (g) | . 42 | 100 | . 16 | . 28 | 50.1 | 4.42 | . 1 | . 26 | . 33 | 11 |
| Carbohydrates (g) | 4.65 | 0 | 8.63 | 28.17 | 2.5 | 0 | 5.57 | 3.75 | 4.64 | 23.6 |
| Calcium (mg) | 5 | . 18 | 20 | 10 | 0 | 17 | 20 | 136 | 5 | 128 |
| Iron (mg) | 1.24 | . 38 | . 22 | 1.2 | 0 | . 99 | 2.38 | 3.57 | . 45 | . 09 |
| Magnesium (mg) | 10 | . 01 | 10 | 12 | 0 | 33 | 40 | 87 | 11 | 14 |
| Phosphorus (mg) | 104 | 1.22 | 33 | 43 | 0 | 295 | 130 | 56 | 24 | 105 |
| Potassium (mg) | 370 | 0 | 157 | 35 | 7.5 | 414 | 212 | 466 | 222 | 199 |
| Sodium (mg) | 4 | . 04 | 3 | 1 | . 5 | 86 | 5586 | 70 | 9 | 80 |
| Zinc (mg) | . 73 | . 06 | . 19 | . 49 | 0 | . 71 | . 43 | . 76 | . 09 | . 69 |
| Copper (mcg) | . 492 | 0 | . 06 | . 069 | 0 | . 099 | . 135 | . 174 | . 074 | . 023 |
| Vitamin C (mg) | 3.5 | 0 | 6.4 | 0 | 0 | 0 | 0 | 9.8 | 19.1 | . 6 |
| Thiamine (mg) | . 102 | 0 | . 042 | . 163 | 0 | . 196 | . 059 | . 095 | . 059 | . 041 |
| Riboflavin (mg) | . 449 | 0 | . 02 | . 013 | 0 | . 073 | . 152 | . 236 | . 048 | . 24 |
| Niacin (mg) | 40116 | 0 | . 148 | 1.476 | 0 | 8.526 | 3.951 | . 49 | . 628 | . 116 |
| Pantothenic Acid (mg) | 2.2 | 0 | . 106 | . 39 | 0 | . 865 | . 376 | . 145 | . 247 | . 581 |
| Vitamin B6 (mg) | . 097 | 0 | . 116 | . 093 | 0 | . 231 | . 2 | . 242 | . 08 | . 048 |
| Vitamin B12 (mcg) | 0 | 0 | 0 | 0 | 0 | 3.46 | 0 | 0 | 0 | . 39 |
| Vitamin A (IU) | 0 | 0 | 0 | 0 | 0 | 136 | 0 | 8190 | 623 | 409 |

Table 2: Nutritional Values of Various Foods per 100g of foodstuff

| Nutrient | Amount |  |
| :--- | ---: | :--- |
| Calories | 8279.12 | kcal |
| Protein | 608.81 | g |
| Fat | 387.60 | g |
| Carbohydrates | 604.48 | g |
| Calcium | 4067.42 | mg |
| Iron | 93.34 | mg |
| Magnesium | 1714.73 | mg |
| Phosphorus | 8488.03 | mg |
| Potassium | 18023.48 | mg |
| Sodium | 8846.38 | mg |
| Zinc | 36009.75 | mg |
| Copper | 6.67 | mcg |
| Vitamin C | 604.06 | mg |
| Thiamine | 6.77 | mg |
| Riboflavin | 10.61 | mg |
| Niacin | 28212.10 | mg |
| Pantothenic Acid | 103.11 | mg |
| Vitamin B6 | 189.81 | mg |
| Viamin B12 | 51.78 | mcg |
| Vitamin A | 95382.93 | IU |

Table 3: Sumo Wrestler Diet

| Nutrient | Amount |  |
| :--- | ---: | :--- |
| Calories | 2000 | kcal |
| Protein | 50 | g |
| Fat | 65 | g |
| Carbohydrates | 300 | g |
| Calcium | 1000 | mg |
| Iron | 18 | mg |
| Magnesium | 400 | mg |
| Phosphorus | 1000 | mg |
| Potassium | 3500 | mg |
| Sodium | 2400 | mg |
| Zinc | 15 | mg |
| Copper | 2000 | mcg |
| Vitamin C | 60 | mg |
| Thiamine | 2 | mg |
| Riboflavin | 2 | mg |
| Niacin | 20 | mg |
| Pantothenic Acid | 10 | mg |
| Vitamin B6 | 2 | mg |
| Viamin B12 | 6 | mcg |
| Vitamin A | 5000 | IU |

Table 4: FDA Recommended Daily Values

