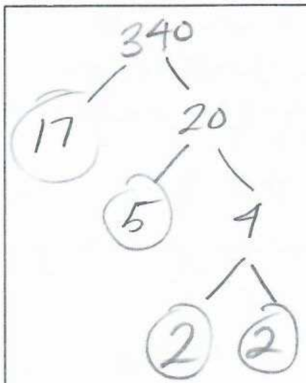
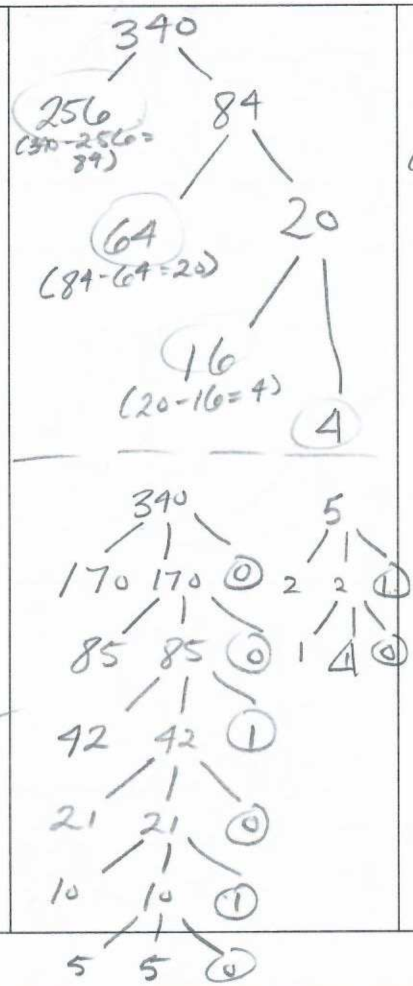
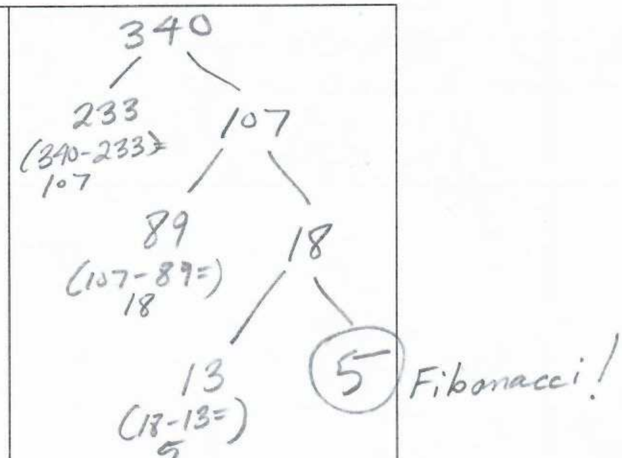
Directions: Show your work! Answers without justification will likely result in few points. I do give partial credit in the event of an incorrect final answer (but good reasoning). Good luck!

a. First few primes: 2 3 5 7 11 13 17 19 23 29 31 37

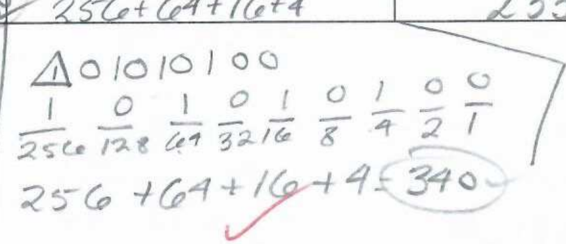
b. First few powers of 2: 1 2 4 8 16 32 64 128 256 512

c. First few Fibonaccis: 1 1 2 3 5 8 13 21 34 55 89 144 233

Problem 1: (18 pts) Your number is 340. For each of the three problems below, draw the appropriate tree – or **show me your work somehow** – and then put your final result in the bottom table, as appropriate for the situation.

Prime Factorization	Binary Factorization	Fibonacci Factorization
 17·5·2·2=340		 Well done

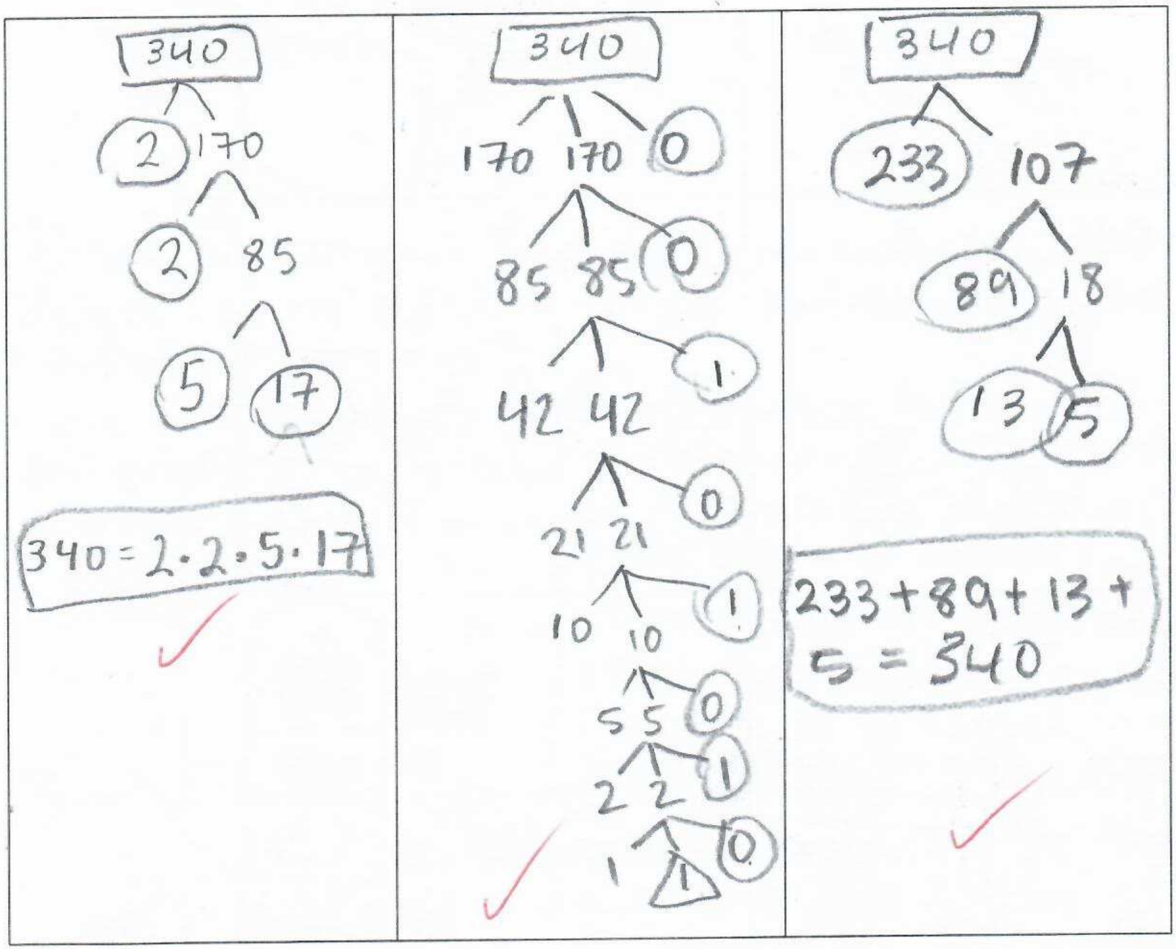
340 = 17·5·2·2=340	340 = 256+64+16+4	340 = 233+89+13+5=340



18

Problem 1: (18 pts) Your number is 340. For each of the three problems below, draw the appropriate tree – or **show me your work somehow** – and then put your final result in the bottom table, as appropriate for the situation.

Prime Factorization	Binary Factorization	Fibonacci Factorization



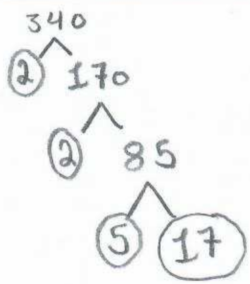
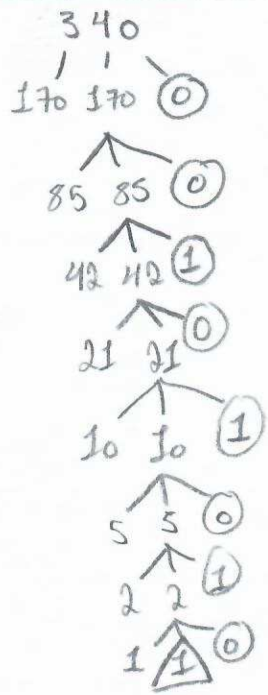
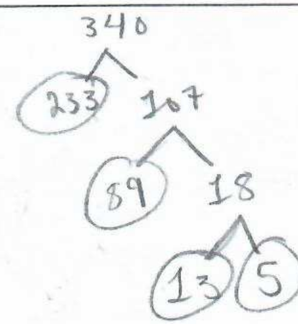
340 = 2·2·5·17	340 = 101010100	340 = 233+89+13+5

1 0 1 0 1 0 1 0 0
256 128 64 32 16 8 4 2 1 =

256 + 64 + 16 + 4 = 340

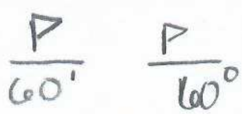
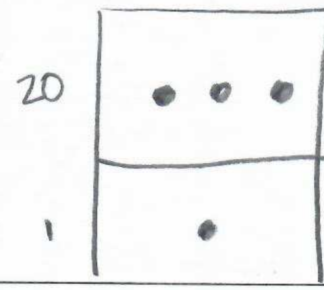
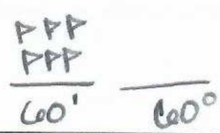
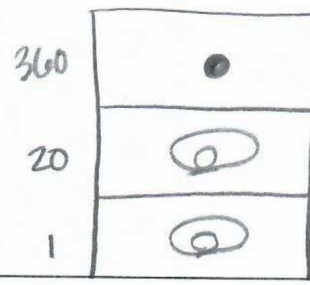
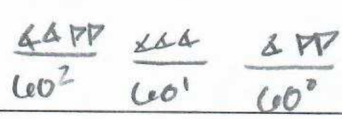
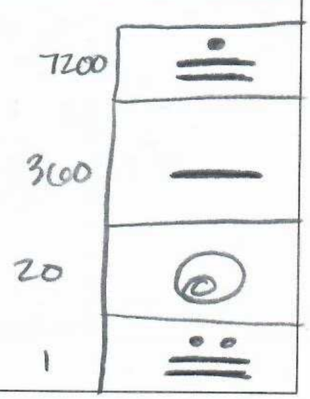
18

Problem 1: (18 pts) Your number is 340. For each of the three problems below, draw the appropriate tree – or **show me your work somehow** – and then put your final result in the bottom table, as appropriate for the situation.

Prime Factorization	Binary Factorization	Fibonacci Factorization
		 340 - 233 = 107 107 - 89 = 18 18 - 13 = 5

340 = 2 · 2 · 5 · 17	340 = 101010100	340 = 233 + 89 + 13 + 5

Problem 2: (18 pts) Write each of the following numbers in Babylonian and in Mayan:

Number	Babylonian	Mayan
61	61 → 1 (60^1), 1 (60^0) ✓ 	61 → 3 (20), 1 ✓ 
360	360 → 6 (60^1) ✓ 	360 ✓ 
81012	81012 → 22 (60^2), 30 (60^1), 12 (60^0) ✓ 	81012 → 11 (7200), 5 (360), 0 (20), 12 (1) ✓ 

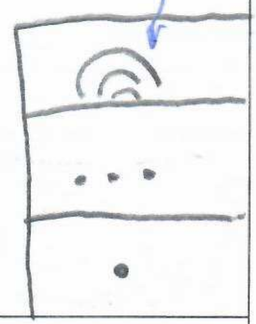
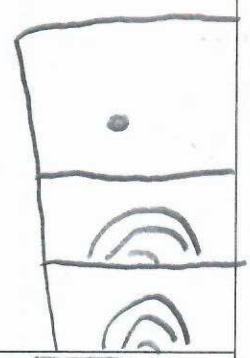
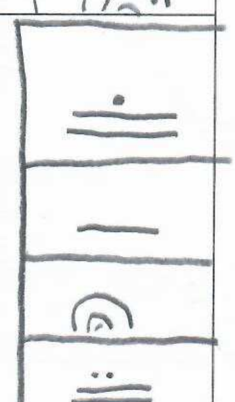
3600 × 22 = 79200; 60 × 30 = 1800

81012 − 79200 = 1812; 1812 − 1800 = 12

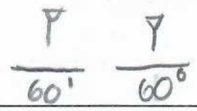
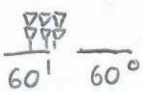
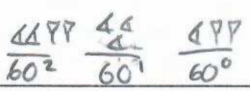
7200 × 11 = 79200; 360 × 5 = 1800

81012 − 79200 = 1812; 1812 − 1800 = 12

Problem 2: (18 pts) Write each of the following numbers in Babylonian and in Mayan:

Number	Babylonian	Mayan
61	60^1 60^0	61 = 3 × 20 = 60, 1 × 1 61 =  show off! :)
360	360 = 60 · 6 60^1	360 = 
81012	81012 = 3600 × 22 = 79,200; 1,812 = 30 × 60 = 1,800; 12 × 1 60^2 60^1 60^0	81012 → =  11 · 7200 = 79200; 1812 = 5 × 360 = 1800; 12 · 1

Problem 2: (18 pts) Write each of the following numbers in Babylonian and in Mayan:

Number	Babylonian	Mayan
61	61 1 · 60 1 · 1 	61 3 · 20 1 · 1 
360	360 6 · 60 	360 1 · 360 
81012	81012 — 12 22 · 3600 1812 12 · 1 30 · 60 	81012 11 · 7200 1812 12 5 · 360 12 · 1 

Problem 3: (20 pts) Short answer. **Do four of the six (write "skip" on the other two).**

a. Draw the first six triangular numbers **as triangles**, and relate them to a game or two.

These numbers are seen in bowling and pool

b. Describe a one-to-one correspondence that was used by Ernie at the Furry Arms Hotel.

A one-to-one correspondence used by Ernie was the correspondance between the counting numbers and the amount of fish ordered.

good

c. Describe how a shepherd might use a one-to-one correspondence to keep track of her sheep (without explicitly counting them).

She might correspond certain types of knots on a string to amounts of sheep, and use strings to record her amounts of sheep.

d. Why does Pascal's name belong on "Pascal's triangle", and why does it **not** belong there?

Pascal's name belongs on the triangle because he contributed greatly to the popularization of the triangle when many people didn't know about it.

His name does not belong on it because many other cultures discovered the triangle before Pascal did.

e. Test to decide whether 277 is prime or not.

1 2 3 5 7 11 13

$\sqrt{277} = 16.294$

277 is prime

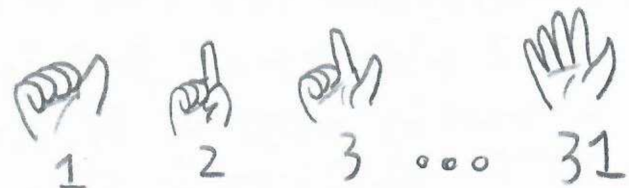
277/2 = 138.5 277/3 = 92.33 277/5 = 55.4 277/7 = 39.57

277/11 = 25.18 277/13 = 21.308 No whole numbers

f. How did Vi Hart use one hand to count from 1 to 31, while binary hand-dancing?

She assigned a power of 2 to each finger, allowing her to count up to 31 in base 2.

Nice! :)



Problem 3: (20 pts) Short answer. **Do four of the six (write "skip" on the other two).**

a. Draw the first six triangular numbers **as triangles**, and relate them to a game or two.

1 3 6 10 15 21 ~~28~~ 28

There are 6 pins in bowling and 15 balls in a game of pool each arranged in a triangle

b. Describe a one-to-one correspondence that was used by Ernie at the Furry Arms Hotel.

one to one correspondence is an association between two sets so that each member of one set has a unique partner in the other set and vice versa such as each order of fish needed at the furry arms hotel was communicated be the use of the word fish, giving one time saying fish for each penguin's order

c. Describe how a shepherd might use a one-to-one correspondence to keep track of her sheep (without explicitly counting them).

Skip

d. Why does Pascal's name belong on "Pascal's triangle", and why does it **not** belong there?

Nice

- Pascals name does <u>not</u> belong there because he did not originally create the triangle
- Pascals name does belong there because he learned the many different uses and secrets of the triangle and documented how to use this piece of Mathmatics including: Probability (as he used in gambling) counting numbers, triangular numbers, tetrahedreal + pentatope numbers as well as the powers of 2 within the triangle he made the triangle known + famous

e. Test to decide whether 277 is prime or not.

Skip

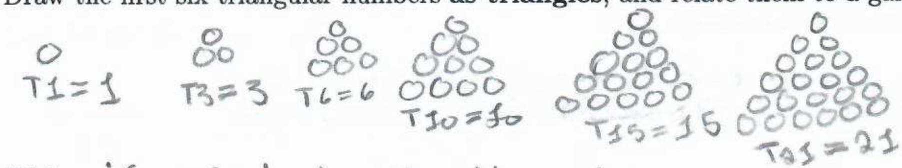
f. How did Vi Hart use one hand to count from 1 to 31, while binary hand-dancing?

Vi Hart labeled her hands with the powers of two and then added the fingers she put down and said the composite such as we did in the fraudini trick with the cards. She labeled her fingers with numbers: 1, 2, 4, 8, 16 so the highest she could go counting on one hand was 31 by adding all her fingers up.

20 Nice work!

Problem 3: (20 pts) Short answer. **Do four of the six (write "skip" on the other two).**

a. Draw the first six triangular numbers **as triangles**, and relate them to a game or two.



T10 is used in bowling, its how the pins are set up. Its also used in pool, how the balls are organized.

b. Describe a one-to-one correspondence that was used by Ernie at the Furry Arms Hotel.

The one-to-one correspondence that was used was to keep track of the amount of fish ordered by the penguins. Since they didn't know counting #s, they would relay the order by saying "fish" as many times as it was ordered. So you know there are as many penguins as fish ordered, without even counting.

c. Describe how a shepherd might use a one-to-one correspondence to keep track of her sheep (without explicitly counting them).

A Shepard might keep as many stones as there are sheep. So if she can see a sheep, a sheep, a sheep, and a sheep she would pick up a stone, a stone, a stone, and a stone. If she lost a sheep she'd get rid of a stone as well. So she knows she has as many sheep as she has stones.

d. Why does Pascal's name belong on "Pascal's triangle", and why does it **not** belong there?

Skip

e. Test to decide whether 277 is prime or not.

$\sqrt{277} \approx 16.64$

Prime, because it doesn't divide evenly by any primes up to 13.

Problem 4: (14 pts) The Great Fraudini, his magic trick, and his game of Nim.

a. (6 pts) The Great Fraudini used six cards in his magic trick.

i. How many numbers from 1 to 63 appear on **exactly three** of them? (Note that this is equivalent to asking how many ways there are of choosing three cards from six.) Explain how you got your answer.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 (20) 15 6 1

1+2+4
1+2+8
1+2+16
1+2+32
1+4+8
1+16+8
1+32+8
1+4+16

1+4+32

$\frac{6!}{3!3!}$ = 20

ii. How many numbers appear on **exactly two** of them? On **exactly four** of them?

C^6_0 C^6_1 C^6_2 C^6_3 C^6_4 C^6_5 C^6_6
1 6 15 20 15 6 1

(15)

$\frac{6!}{2!4!}$

b. (12 pts) You're playing a game of Fraudini Nim against the Great Fraudini himself. (Don't let him take your points!) Assume that you each **should** play with the perfect strategy, as described in class.

i. You are looking at 29 counters, and decide to pick up 8 of them. What will the Great Fraudini do, and why?

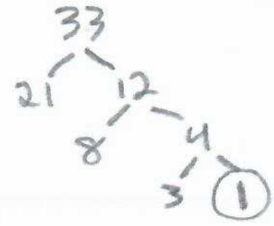
FRAUDINI IS NOW LOOKING AT A FIBONACCI NUMBER, SO HE WILL TAKE ONE TO SLOW THE GAME DOWN

ii. In the course of another game, the Great Fraudini took 1 counter from 24, leaving you staring at 23. Who is going to win this game, and why?

IF I TAKE 2 THIS TURN THEN FRAUDINI IS LOOKING AT A FIBONACCI NUMBER SO I AM IN A GOOD SPOT TO WIN

iii. The starting number of counters is 34. Do you go first? Why or why not? If you do go first, and take 1 counter, how many counters will Fraudini take? (How do you know?)

SINCE 34 IS A FIBONACCI NUMBER I WOULD ALREADY BE OFF TO A BAD START. IF I HAD TO GO FIRST AND TAKE 1 LEAVING FRAUDINI WITH 33, HE WOULD TAKE 1 AS ITS THE SMALLEST IN THE SUM OF UNIQUE FIBONACCI NUMBERS



18

Problem 4: (14 pts) The Great Fraudini, his magic trick, and his game of Nim.

a. (6 pts) The Great Fraudini used six cards in his magic trick.

i. How many numbers from 1 to 63 appear on **exactly three** of them? (Note that this is equivalent to asking how many ways there are of choosing three cards from six.) Explain how you got your answer.

$$C^6_3 = 20 = \frac{6!}{3!(6-3)!} = \frac{6\cdot5\cdot4\cdot3\cdot2\cdot1}{(3\cdot2\cdot1)(3\cdot2\cdot1)} = \frac{120}{6} = \boxed{20}$$

Or got to 6th row on Pascal's Δ and 3 over

ii. How many numbers appear on **exactly two** of them? On **exactly four** of them?

2 cards:

$$C^6_2 = 15 = \frac{6!}{2!(6-2)!} = \frac{6\cdot5\cdot4\cdot3\cdot2\cdot1}{(2\cdot1)(4\cdot3\cdot2\cdot1)} = \frac{30}{2} = \boxed{15}$$

Pascal's Δ: 6th counting number row, 2 in

4 cards:

$$C^6_4 = \frac{6!}{4!(6-4)!} = \frac{6\cdot5\cdot4\cdot3\cdot2\cdot1}{(4\cdot3\cdot2\cdot1)(2\cdot1)} = \frac{30}{2} = \boxed{15}$$

Pascal's Δ: 6th counting number row, 4 in

Problem 5: (16 pts)

- (6 pts) Draw the complete graphs K_2, K_3, K_4, and K_5. How many edges does each have? Identify the pattern.

K2 = 1 K3 = 3 K4 = 6 K5 = 10 ✓

the pattern is the number of points multiplied by one less than the number of points, divided by 2. $\left(\frac{n \cdot (n-1)}{2}\right)$ ✓

- (4pts) Translate the following number, expressed in binary (or base 2), into our ordinary base 10 number:

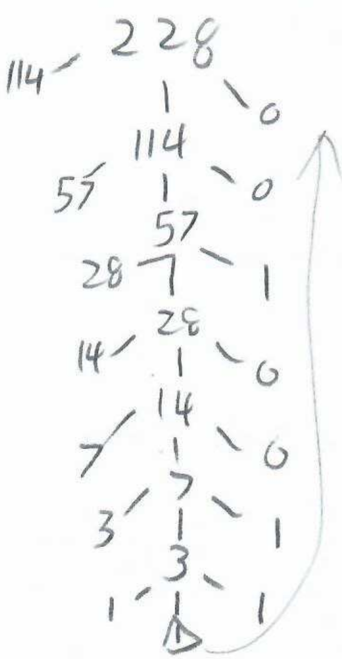
$$10110011_2$$

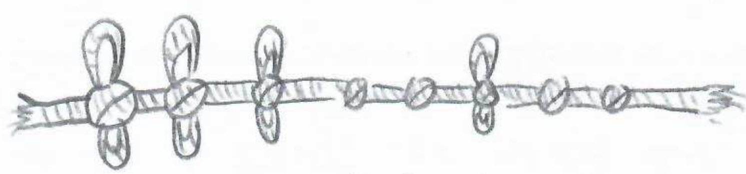
128 + 32 + 16 + 2 + 1

179 ✓

- (6 pts) **Use a ternary tree** to demonstrate primitive counting for your flock of 228 sheep. You are to write the string of zeros and ones that will communicate the size of your flock to the King, as we did in class.

11100100





= 228 Sheep

Beautiful!

+2

Problem 5: (16 pts)

- (6 pts) Draw the complete graphs K_2, K_3, K_4, and K_5. How many edges does each have? Identify the pattern.

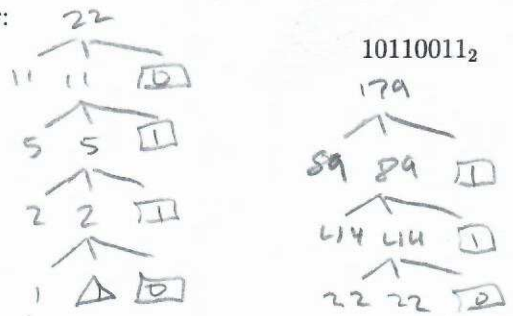
1 3 6 10

IT IS TRIANGULAR NUMBERS

1, 3, 6, 10, 15.

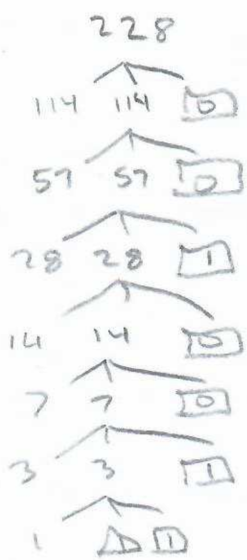
- (4pts) Translate the following number, expressed in binary (or base 2), into our ordinary base 10 number:

10110011_2



179

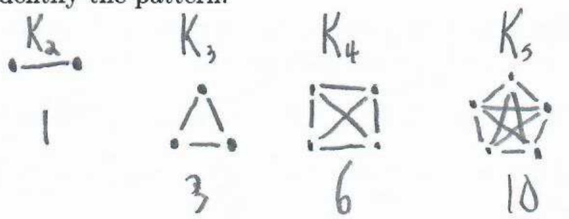
- (6 pts) **Use a ternary tree** to demonstrate primitive counting for your flock of 228 sheep. You are to write the string of zeros and ones that will communicate the size of your flock to the King, as we did in class.



11100100

Problem 5: (16 pts)

- (6 pts) Draw the complete graphs K_2, K_3, K_4, and K_5. How many edges does each have? Identify the pattern.



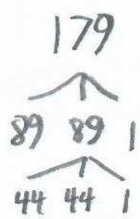
The triangular numbers

✓

- (4pts) Translate the following number, expressed in binary (or base 2), into our ordinary base 10 number:

$\Delta 0110011_2$

good check!



179

✓

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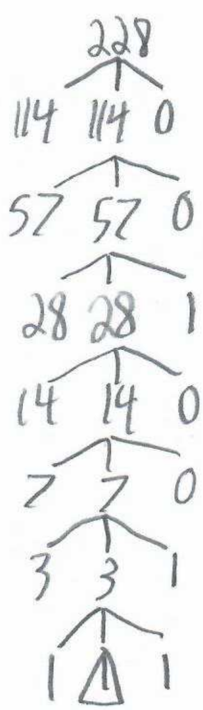
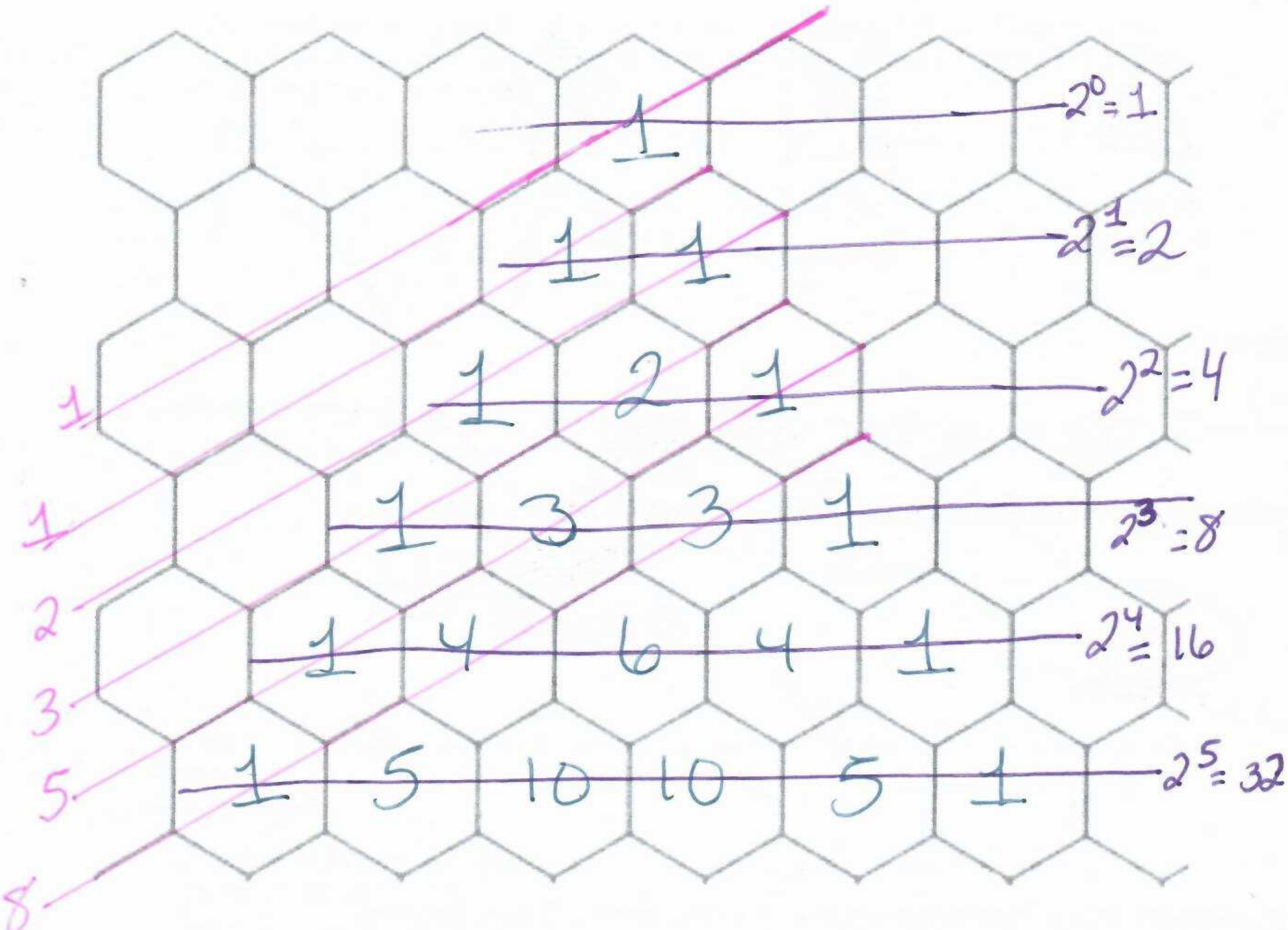


Δ1100100

✓

Problem 6: (14 pts)

a. (4 pts) use this hexagonal grid to create Pascal's triangle, starting down from a "1" in the top row, center:



b. (6 pts) Demonstrate, using the figure above, exactly how the

- powers of two and
- Fibonacci numbers

appear in Pascal's triangle **in a systematic way**.

Powers of 2 are the sum of each row as shown above.

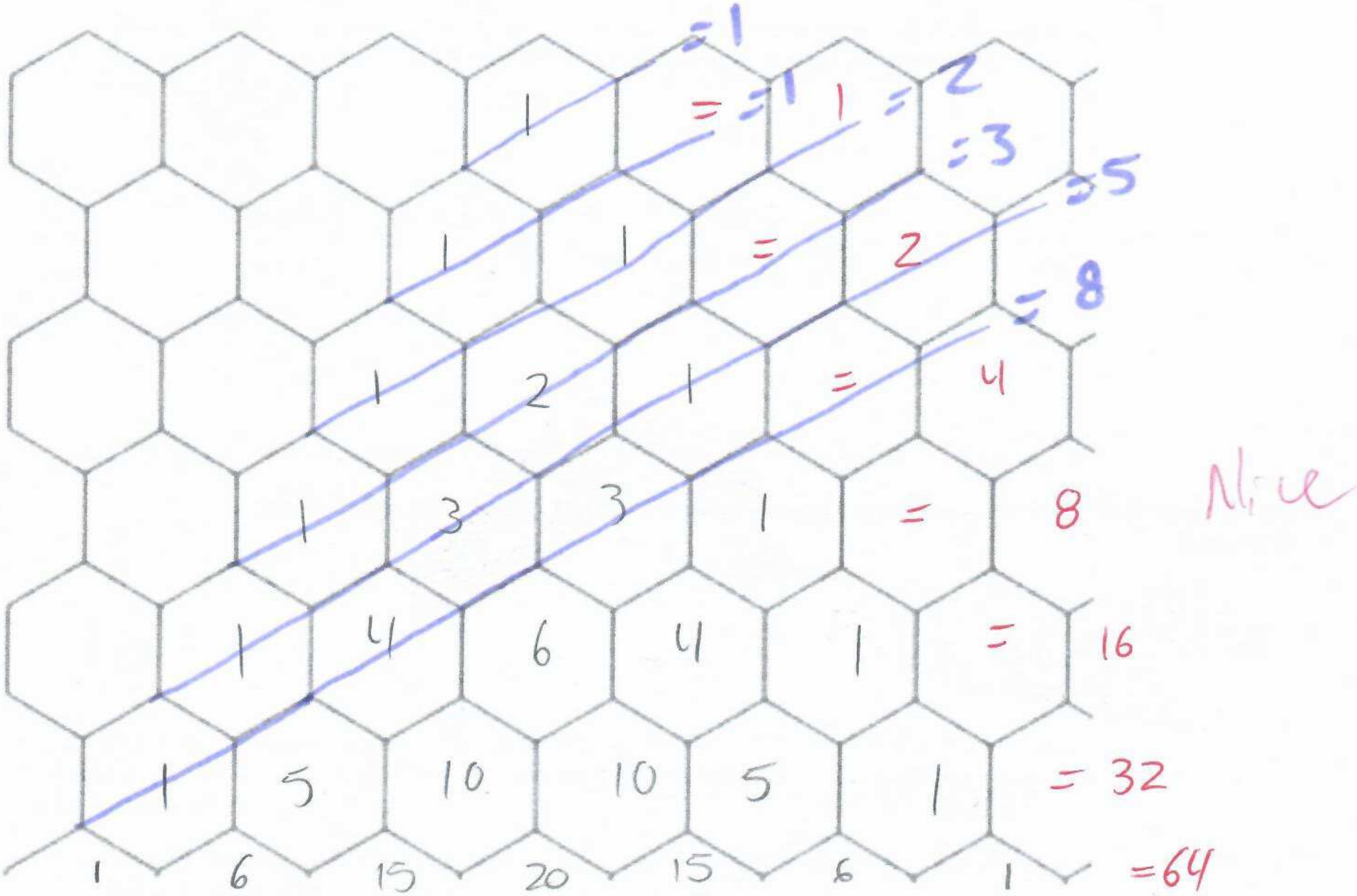
- Fibonacci #s are shown as the sum of the numbers that are bisected on a line when drawn across the vertices as shown above.

c. (4 pts) You can invite three of your five friends to a concert. In how many ways might you make the choice? Explain your answer.

10 - Go to the row where 5 is the counting number, or number in the 2nd to left position. Count from the left, 4 places in. This illustrates the possible ways to choose 0, 1, 2, and then 3 friends as combinations.

Problem 6: (14 pts)

a. (4 pts) use this hexagonal grid to create Pascal's triangle, starting down from a "1" in the top row, center:



b. (6 pts) Demonstrate, using the figure above, exactly how the

- powers of two and red
- Fibonacci numbers purple

appear in Pascal's triangle **in a systematic way**.

c. (4 pts) You can invite three of your five friends to a concert. In how many ways might you make the choice? Explain your answer.

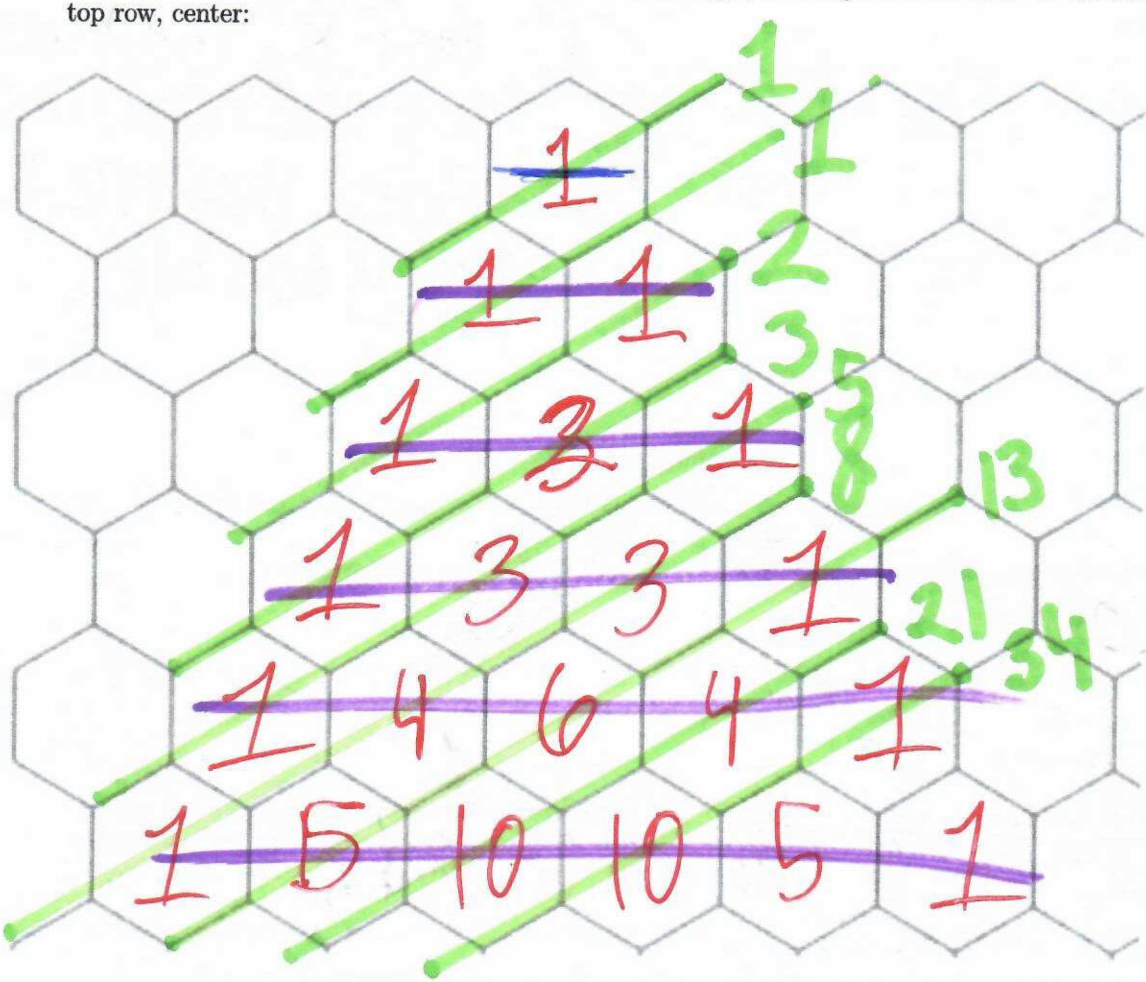
$$C^5_3 = \frac{5!}{3!(5-3)!} = \frac{5\cdot4\cdot3\cdot2\cdot1}{3\cdot2\cdot1\cdot2\cdot1} = \frac{20}{2} = 10$$

you can also go down 5 in 3 on Pascals triangle

10 ways

Problem 6: (14 pts)

a. (4 pts) use this hexagonal grid to create Pascal's triangle, starting down from a "1" in the top row, center:



b. (6 pts) Demonstrate, using the figure above, exactly how the

- powers of two and Powers of 2
- Fibonacci numbers Fibs

appear in Pascal's triangle **in a systematic way.**