Average Value of a Function

Sea Ice extent has varied considerably over the past decade, as illustrated in Figure 1:

Figure 1: This image (source) is based on data (source) obtained from the Japan Aerospace Exploration Agency.



Here is the website's description of the data:

- Data from June 2002 to the present are included.
- The format is "month, day, year, extent (unit: square km)".
- Number "-9999" represents that we couldn't conduct the observation during the period for the reason that the satellite went into constrained operation mode or stand-by mode to avoid harmful effects by meteor showers and solar flares.

The data is given almost daily for much of the period, but there are a fair number of missing values (as described). If you wish, visit http://www.ijis.iarc.uaf.edu/en/home/seaice_extent.htm to see how sea ice extent is defined.

In order to give us something to work with in this lab, I averaged the daily data over each month in a year, to give the results (Table 1) for three of the years:

	2003	2007	2010
January	1.35326E+7	1.29036E+7	1.28771E+7
February	1.43623E + 7	1.36982E + 7	$1.37691E{+}7$
March	1.47057E + 7	$1.37179E{+}7$	$1.42785E{+7}$
April	1.36569E + 7	1.30360E + 7	$1.38357E{+}7$
May	1.21824E + 7	$1.19415E{+7}$	$1.19959E{+}7$
June	1.09058E+7	$1.04981E{+7}$	1.00305E+7
July	8.84840E+6	7.66539E + 6	7.90460E + 6
August	6.77090E+6	5.30139E + 6	5.96827E + 6
September	6.12647E + 6	4.38052E + 6	5.09709E + 6
October	7.54304E+6	6.00201E + 6	6.91923E + 6
November	9.91880E+6	9.25249E + 6	9.24517E + 6
December	1.18946E + 7	1.14608E + 7	$1.11049E{+}7$

Table 1: We might associate the following values with the midpoint of each month (since they're averages for all data values available for a given year and month).

Questions:

- 1. If we integrated over a given year, what would be the sense of the answer? What would it represent physically?
- 2. Which approximation techniques that we have described could be used to perform the integration?
- 3. How might one compute the **average** sea ice extent, using the methods of this section? Do so, and describe its meaning. What do you conclude about average sea ice extent for the three different years?
- 4. Did you consider the difference in month length in your calculations? Should you?