Calculus I: General Concepts

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1 Functions

- The function zoo: polynomial, rational, trig, power, root, etc.
- domain, range
- symmetry (even, odd)
- asymptotes (vertical, horizontal, slant)
- increasing, decreasing
- concavity
- continuities and discontinuities (connectedness)
- differentiability (smoothness)
- one-to-one, invertible (pass horizontal and vertical line tests)
- compositions of functions
- transformations (shifts and stretches) of functions

2 Limits

- limit laws
- tangent versus secant lines
- limits at infinity, and infinite limits
- continuity and discontinuity
- Intermediate value theorem
- pinching or squeeze theorem

3 Derivatives and differentiation

- The limit definition of the derivative function (the most important definition in calculus!):

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- relationship between the size and sign of the derivative of f and the shape of the graph of f
- sum, difference, product, quotient rules
- chain rule (derivatives of compositions)
- implicit differentiation
- related rates problems
- optimization problems
- antiderivatives
- higher derivatives
- linearization
- Mean value theorem (Rolle's theorem on a tilt)

4 Optimization

- Setting up the problem is the hard part! Draw pictures....
- extrema (maxima, minima; local and global)
- first and second derivative tests
- check endpoints!

5 Integrals and Integration

- Motivation (the beginning, but not the end!): areas
- using rectangles via approximation rules to estimate areas (left, right, midpoint, trapezoidal – ultimately Simpson's rule).
- When you have two estimates for a quantity, you have a third – some average of the two. The average may be weighted, depending on the confidence you have in the two estimates.
- taking a limit and defining the area exactly

- **Fundamental Theorem of Calculus** (tying derivates together with integrals): Suppose f is continuous on $[a, b]$.

    1. If $g(x) = \int_a^x f(t)\, dt$, then $g'(x) = f(x)$.

    2. $$\int_a^b f(x)dx = F(b) - F(a)$$

       where F is any antiderivative of f.

- substitution rule (the chain rule in reverse)

- integrals as more general than simply areas: so

  $$S = \int dS$$

  means that in computing quantity S (area, length, volume, time, dollars, etc.) we simply add up lots of tiny amounts (infinitesimals) of S. Examples:

  - areas between two curves
  - An example of an important extension of integrals is volume calculations:

    $$V = \int dV = \int A(x)dx = \int C(x)dxh(x)$$

    We either slice into cross-sections of known area $A(x)$, or we chop into cylinders of known circumference $C(x)$ and height $h(x)$.
  - Here's the sort of application one might do someday as an engineer. The time to arrive at State Route 126 from Exit 3 on I-75 is

    $$T = \int dT = \int_{\text{Exit 3 on I--75}}^{\text{State route 126}} \frac{dx}{v(x)}$$

    where $v(x)$ is the average automobile velocity at every point along the road from point a to point b. (dt was calculated using the DiRT formula - don't forget the dirt formula!). To compute a time, we add up lots of little times dT, which are computed from an instantaneous velocity over a very short distance dx.

- using symmetry to help calculate integrals

- definite versus indefinite integrals