

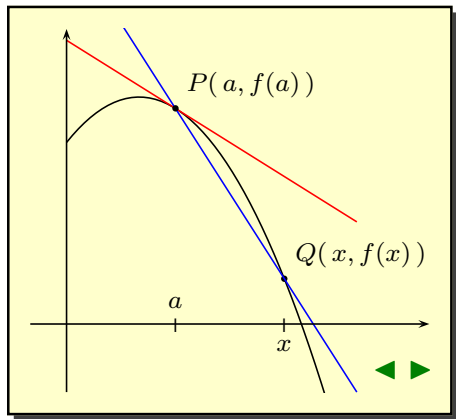
The AcroT_EX Web Site, 2000

A Slide Show
Demonstrating the
Tangent Line Problem

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The Department of Mathematics
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The University of Akron, Akron, OH

Tangent Line Problem

Problem: Given a point $P(a, f(a))$, we want to define and calculate the **slope** of the line tangent the graph at P .



- Choose a point x near a and plot $Q(x, f(x))$.
- Draw the *secant line* through P and Q . The slope of this secant line is

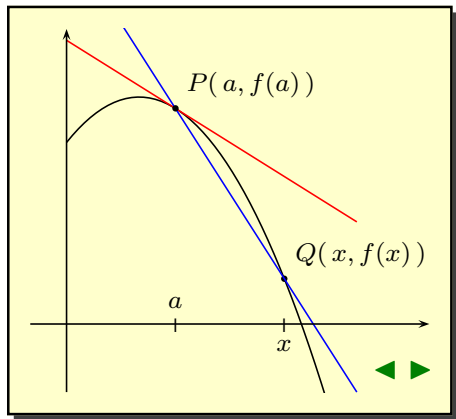
$$m_{\text{sec}} = \frac{f(x) - f(a)}{x - a}$$

Example: $f(x) = 5 - (x - 1)^2$ and $a = 1.5$.

x	3							
m_{sec}	-2.5							

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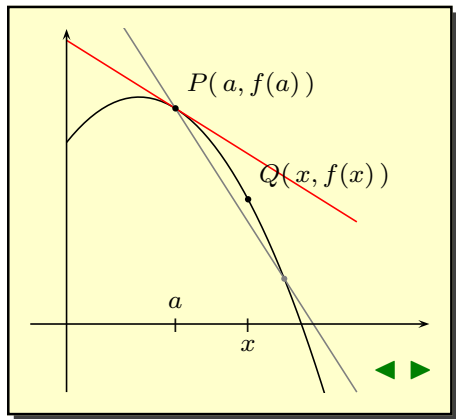
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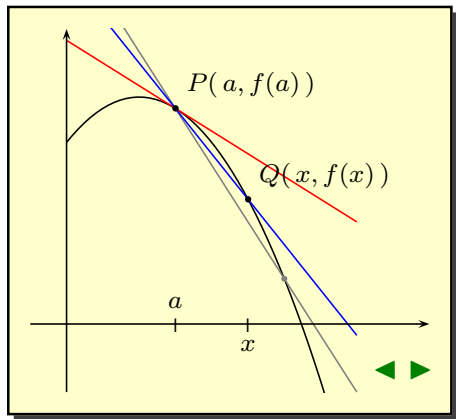
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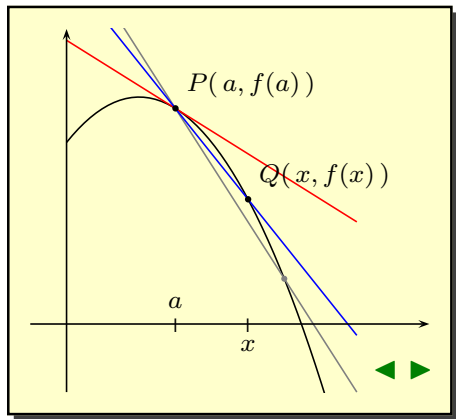
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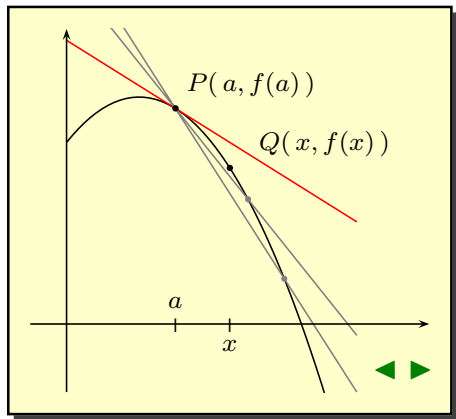
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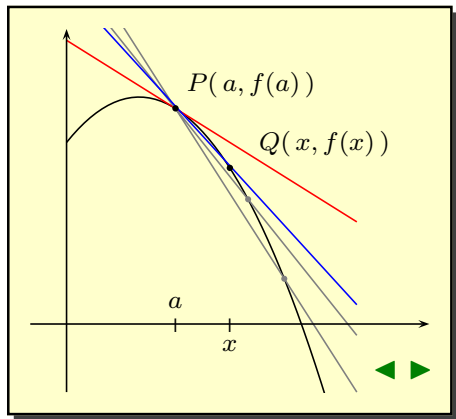
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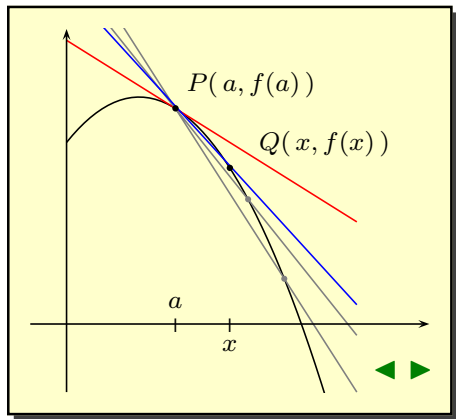
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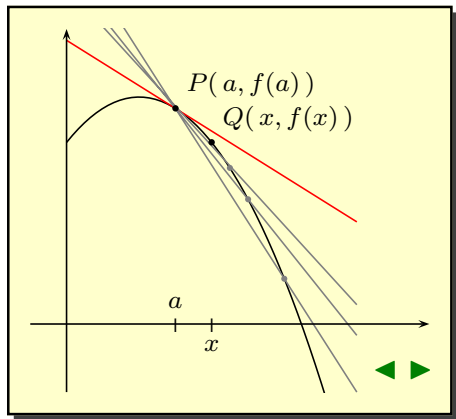
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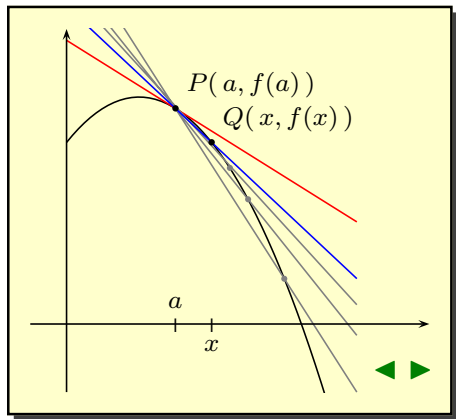
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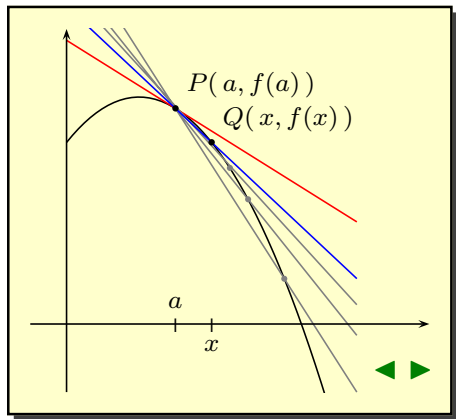
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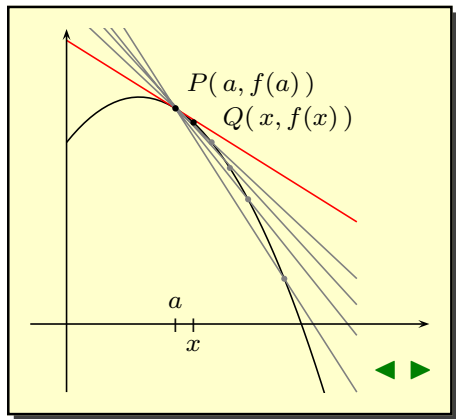
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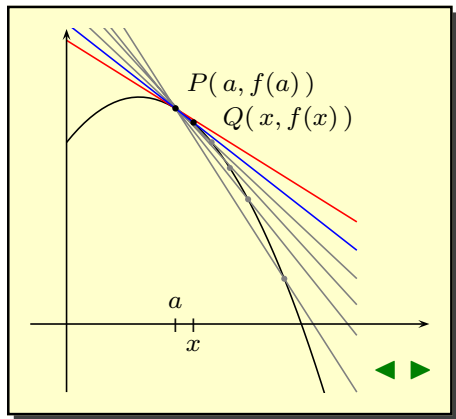
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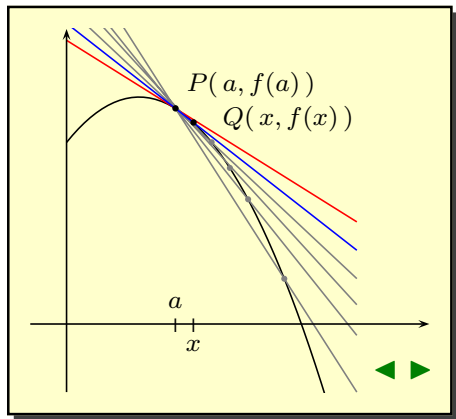
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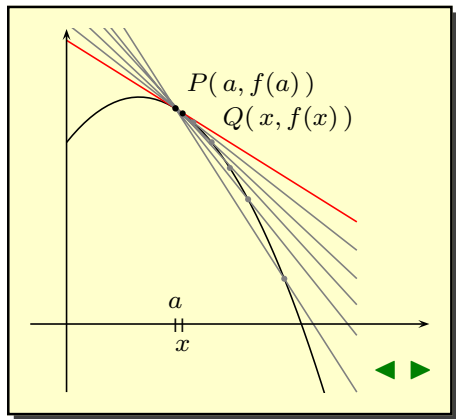
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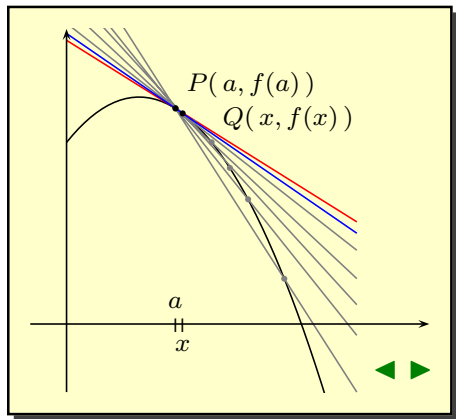
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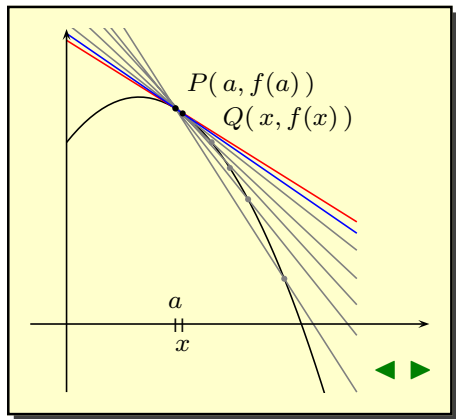
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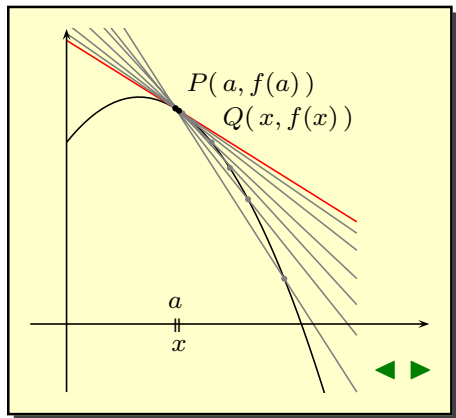
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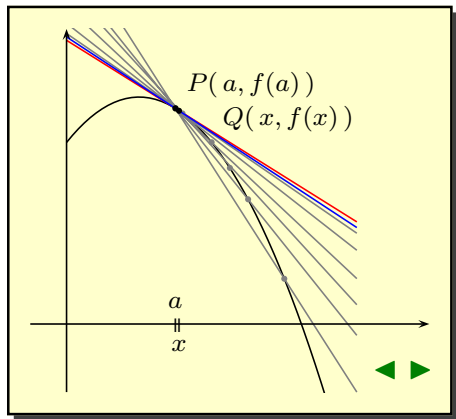
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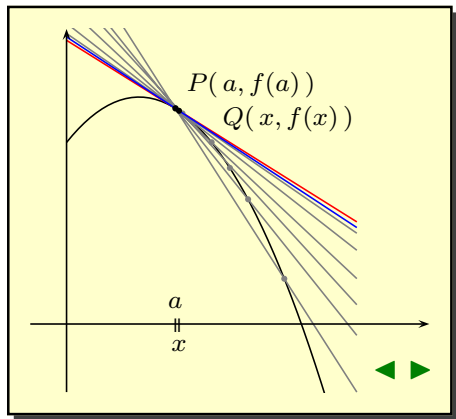
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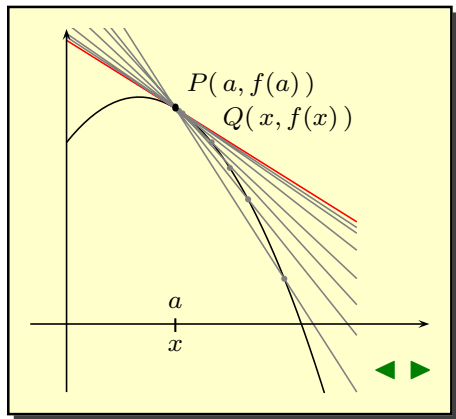
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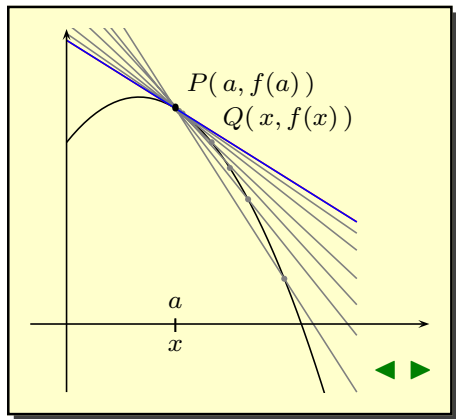
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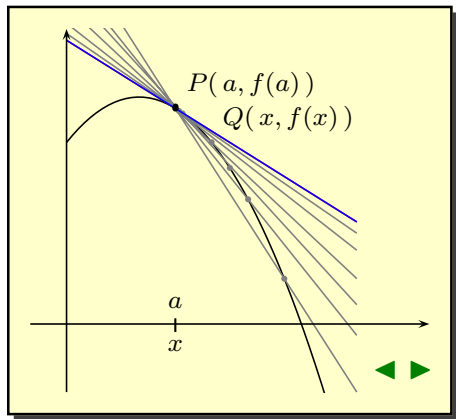
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- Repeat.
- Continue ...

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Discussion

Example: $f(x) = 5 - (x - 1)^2$ and $a = 1.5$. As we choose values of x getting closer and closer to $a = 1.5$, the corresponding secant lines rotate around the point P and become more and more “tangent-like”. Therefore, it is not too surprising that the slopes of these secant lines are approaching a value we would want to call “the slope of the line tangent to the graph at P ”.

There are more calculations for those who want to see more.

$x < 1.5$		$x > 1.5$	
x	m_{sec}	x	m_{sec}
1	-0.5	2	-1.5
1.4	-0.9	1.6	-1.1
1.45	-0.95	1.55	-1.05
1.49	-0.99	1.51	-1.01
1.499	-0.999	1.501	-1.001
1.4999	-0.9999	1.5001	-1.0001
1.49999	-0.99999	1.50001	-1.00001

The values of m_{sec} appear to be getting close and closer to -1 . In this case, we write:

$$\lim_{x \rightarrow 1.5} m_{\text{sec}} = -1$$