Calculus A (mat128): General Concepts

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1 Functions

- The function zoo: polynomial, rational, trig, power, root, etc.
- Linear functions are the most important in calculus, including tangent lines (best represented using point-slope form).
- domain, range
- symmetry (even, odd, or symmetry displaced)
- asymptotes (vertical, horizontal, slant, limits at infinity)
- increasing, decreasing
- concavity - up like a bowl, down like an umbrella
- continuity and discontinuities (connectedness versus holes or jumps)
- differentiability (smoothness)
- compositions of functions
- transformations (shifts and stretches) of functions

2 Limits

- limit laws
- tangent versus secant lines
- limits at infinity, and infinite limits
- continuity and discontinuity
- Intermediate value theorem
- pinching or squeeze theorem

3 Derivatives and differentiation

- The limit definition of the derivative (the most important definition in calculus!):
  - at a point:
$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$
  - function:
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

  which represents the slope of the tangent line to the graph at either $x = a$ or x in general.
- relationship between the size and sign of the derivative of f and the shape of the graph of f
- sum, difference, product, quotient rules
- chain rule (derivatives of compositions)
- trig rules
- higher derivatives – e.g. inflection points, jerk, etc.
- linearization and differentials
- implicit differentiation
- related rates problems
- anti-derivatives
- Newton's method (finding roots of functions)

4 Optimization

- Setting up the problem is the hard part! Draw pictures....
- extrema (maxima, minima; local and global)
- Extreme value theorem
- Fermat's theorem
- first and second derivative tests
- check endpoints!

5 Anti-derivatives

- prepare us for the second half of univariate calculus: the integral calculus.
- Solving differential equations by staring at them until the solution comes to you.
- "Thinking backwards" (e.g. the anti-derivative of x^n is $\frac{x^{n+1}}{n+1}$)
- There are an infinite number (a family) of anti-derivatives for a function; but each differentiable function has a unique derivative. The most general anti-derivative of function $f(x)$ is any particular anti-derivative ($F(x)$) plus an arbitrary constant: $F(x) + C$.