## GROUP WORK 3, SECTION 4.1 <br> Two Easy Pieces

Let $f(x)$ be the piecewise-continuous function graphed below.


1. Compute $L_{2}$ and $R_{2}$, the left-endpoint and right-endpoint Riemann sum approximations for $\int_{0}^{4} f(x) d x$ with two subintervals. How do you think these values compare to the actual value of the integral?
2. Now compute $L_{4}$ and $R_{4}$. Which appears to be the best approximation to the actual value of the integral?
3. Now compute $L_{5}$ and $L_{6}$. Which do you think is closer to the actual value?
4. The values of $L_{9}, L_{10}$, and $L_{11}$ are given in the following table, along with the actual value of the integral.

| $L_{9}$ | 7.173 |
| :--- | :--- |
| $L_{10}$ | 6.560 |
| $L_{11}$ | 7.149 |
| $\int_{0}^{4} f(x) d x$ | 7 |

Can you explain why $L_{10}$ is a poorer approximation than $L_{9}$ and $L_{11}$ ?
5. Do you think a similar pattern holds for $L_{14}, L_{15}$, and $L_{16}$ ? Does this pattern also hold for higher values of $n$ ?

6 What do you think will happen to this discrepancy as $n \rightarrow \infty$ ?

