# MAT128 Final (Fall 2015)

**Name:**

**Directions**: You **must** skip one of problems 7-11 (write "skip" clearly on the one skip). Show your work! Your written work allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

**Problem 1:** Use the limit definition of the derivative to find $f'(x)$, when $f(x) = 2x^2 + 1$.

**Problem 2:** Use standard differentiation rules to calculate the derivative of $f(x) = \dfrac{\sqrt{x}\sin(x^2)}{x+1}$.

Justify each step.

**Problem 3:** Given $f(x) = \sin(\pi x) - 3x$. Find

a. the general anti-derivative $F(x)$ of $f(x)$.

b. that particular anti-derivative of $f$ whose value at $x = 2$ is 5.

**Problem 4:** Study and graph the function $f(x) = \dfrac{6x^2}{x^2+1}$. Point out the usual important features of the function.

a. What kind of function is $f$, and where is it defined?

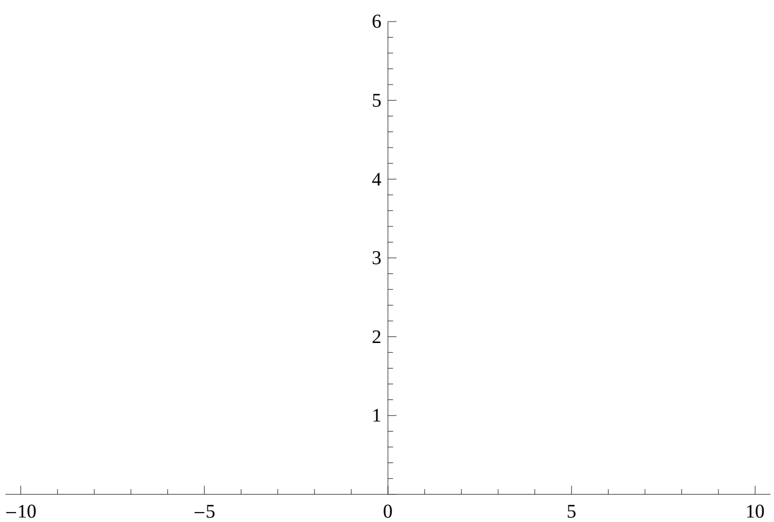
b. What are the function's special properties?

c. Find critical points of $f$.

d. Find and classify extrema of $f$.

e. Find asymptotes of $f$.

f. Find inflection points of $f$.



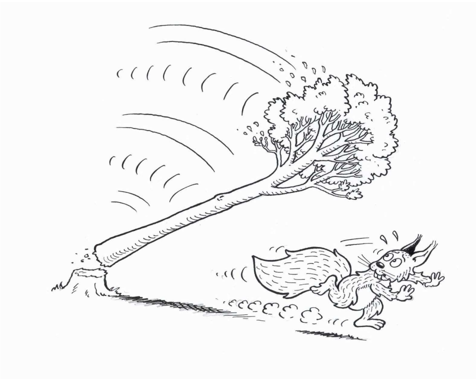
**Problem 5:** The radius of a sphere is given as $r = 10$ cm, but with a possible error of as much as $\pm 0.1$ cm.

The volume formula of a sphere is $V(r) = \frac{4}{3}\pi r^3$; its surface area is $A(r) = 4\pi r^2$.

a. Use differentials to **estimate** the maximum possible error in the volume and surface area of this sphere, based on the stated potential error in the radius.

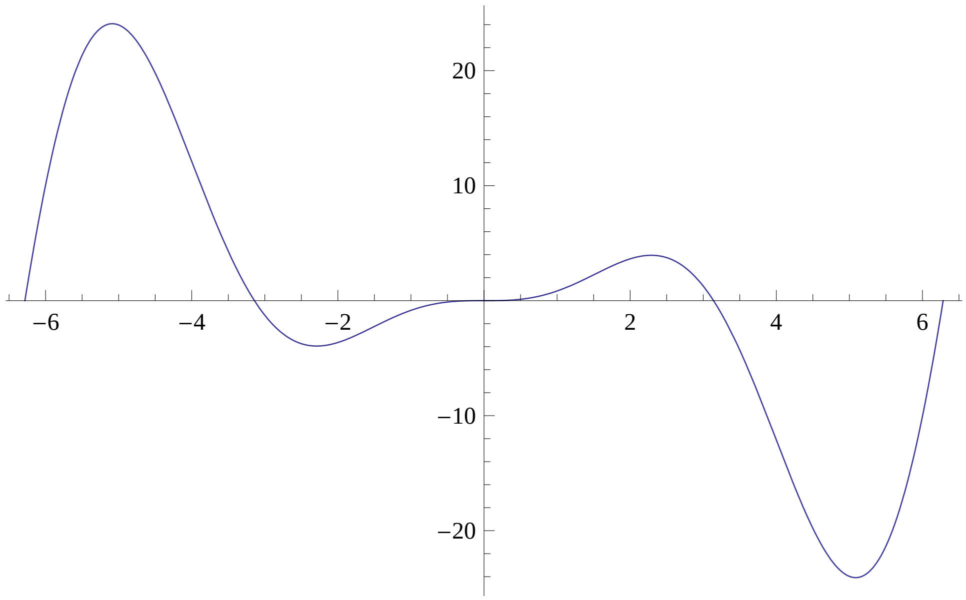
b. What are the **actual** worst errors that could result ($\Delta V$ and $\Delta A$)? How well did your estimates perform?

**Problem 6:** A straight, 100-foot tall tree is falling, cut cleanly at ground level. Something like this figure (but maybe you'd like to clean it up a little to the right of the figure):



As the tree falls, the angle it makes with the ground changes from $90^o$ ($\frac{\pi}{2}$ radians) to $0^o$. At the moment when the angle is $45^o$ ($\frac{\pi}{4}$), the angle is changing at a rate of $\frac{\pi}{6}$ radians per second. How fast is the treetop's height (from tip straight down to the ground) changing at that moment?

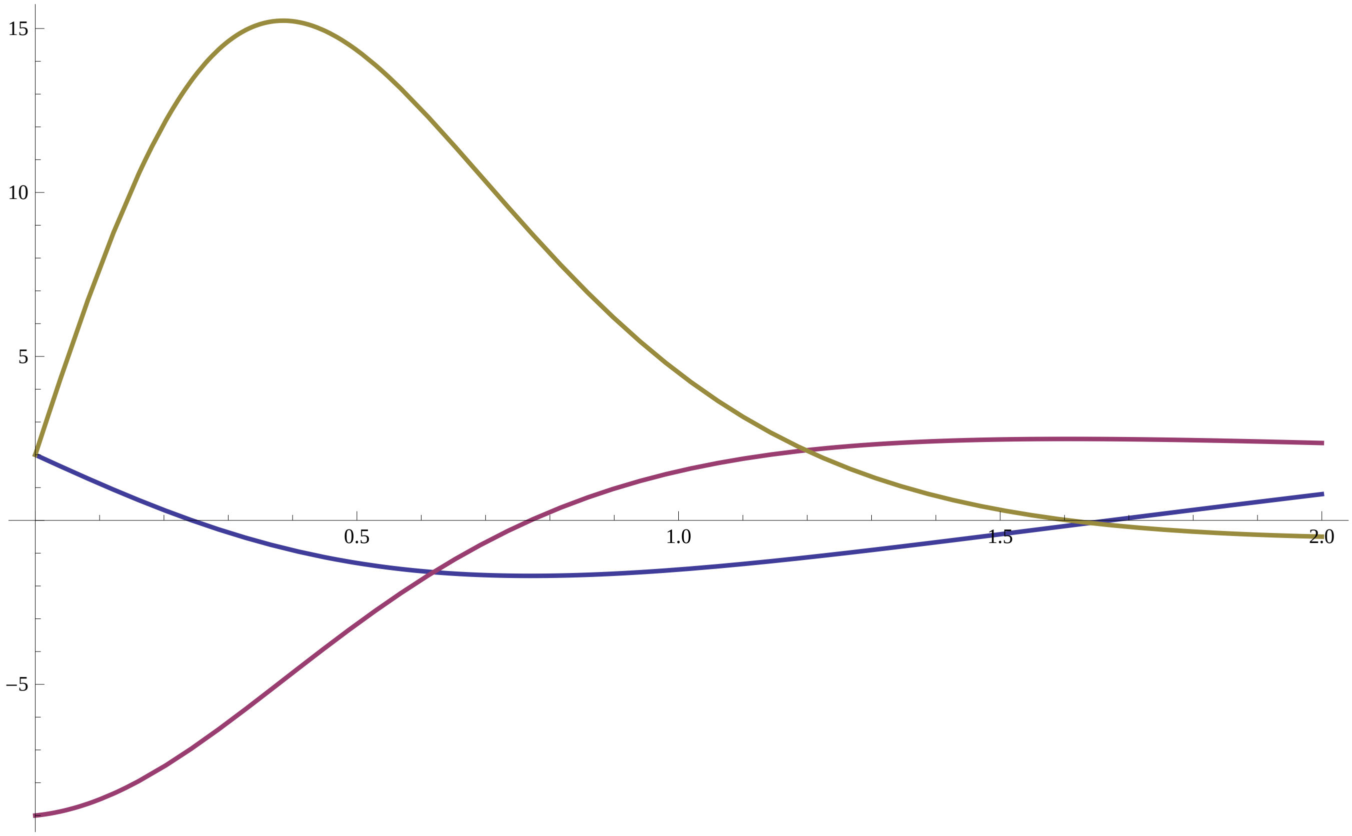
**Problem 7:** Let $f(x) = x^2 \sin(x)$, graphed below:




a. Sketch the graph of the derivative $f'(x)$ on the same axes.

b. Find the equation of the tangent line to the curve at $x = -\pi$, and add it to your graph.

**Problem 8:** With reasons, determine which of the three graphs in the figure is the function, which its derivative, and which its second derivative: Clearly indicate which is which!



**Problem 9:** Short answers:

a. Give a technically correct definition of a function.

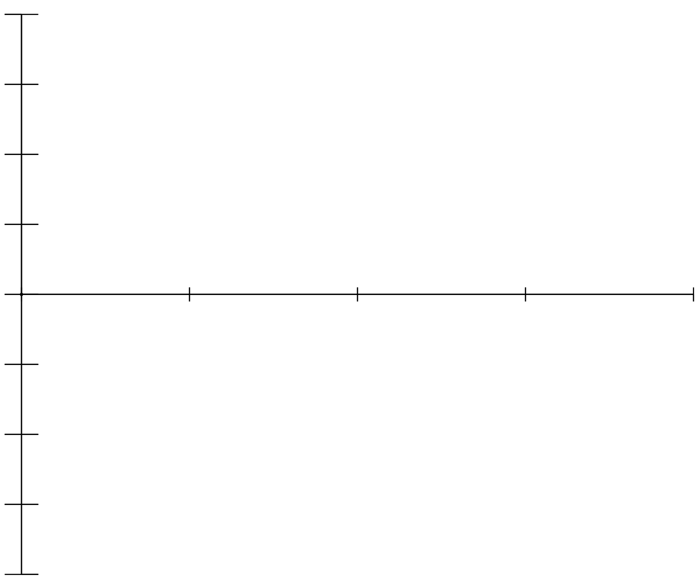
b. Describe accurately the relationship between continuity and differentiability. (Figures are encouraged.)

c. Draw an illustration of the pinching (or squeeze) theorem.

**Problem 10:** Given the following data (height in meters, time in seconds):

| t | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| h(t) | -2 | 1 | 3 | 4 | 3 |
| average h'(t) | NA | | | | |

a. Graph the data from the table on the axes below (label!), and use the data to estimate the **average rate of change** over each second. Add your estimates to the table above (where each answer in the box represents the average rate of change over one of the four seconds).



b. Use the data in the table above and an appropriate secant line to estimate the time rate of change of $h$ at $t = 2$. Add your secant and tangent lines to the graph.

**Problem 11:** Let $f$ be a continuous function on a closed and bounded interval $[a, b]$.

Explain what you can legitimately conclude from each of the following (consider each of these by itself, without regard for the others):

a. The extreme value theorem

b. The intermediate value theorem

c. $f$ is differentiable, and the derivative of $f$ changes sign at $x = c$.

d. $f$ is twice-differentiable, and the second derivative of $f$ changes sign at $x = c$.