MAT225 Test 2 (Fall 2005): Sections from 2.4 to 4.6

Name:

Directions: Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1 (10 pts) Perform the LU decomposition on the matrix

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 5 & 3 \\ 1 & 3 & 3 \end{bmatrix}$$

Problem 2 (10 pts). Consider

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -2 & -4 & -6 \end{bmatrix}$$

1. Find the dimensions of the null and column spaces of the matrix

2. Find bases for each of these two spaces.

Problem 3 (10 pts). The coordinates of $\overline{x}$ in the standard basis are $\overline{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

1. Express $\overline{x}$ in the basis $B = \{\overline{v_1}, \overline{v_2}\} = \{\begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \end{bmatrix}\}$

2. How do we know that the set $\{\overline{v_1}, \overline{v_2}\}$ forms a basis of $\mathbb{R}^2$?

Problem 4 (10 pts). Consider the vector space P^2 of polynomials of degree at most 2.

1. Show that the set of vectors $\{1, x, x^2\}$ is a basis for this space.

2. Differentiation is a linear transformation of P^2 into P^2. Describe the null-space and the column space of this transformation.

Problem 5 (10 pts). Determine whether the sets of vectors of the following forms generate subspaces of $\mathbb{R}^3$ or not. If the expressions represent subspaces, give example spanning sets; otherwise, explain the problem.

1. $\begin{bmatrix} s-1 \\ t-1 \\ u \end{bmatrix}$

2. $\begin{bmatrix} s-t \\ t \\ 2 \end{bmatrix}$

Problem 6 (10 pts).

1. Give a geometric meaning to the determinant in $\mathbb{R}^n$.

2. What is the determinant of a rotation matrix?

3. What is the determinant of the matrix $2I_n$ (where I_n is the n x n identity matrix)?

Problem 7 (10 pts). One partition of matrices we've considered in class is to think of a matrix A as a bunch of column vectors:

$$A = [\overline{v_1}\ \overline{v_2}\ \dots\ \overline{v_n}]$$

1. What will A^T look like in partitioned form?

2. Write AA^T as a sum of rank-one outer-products (that is, products of the form $\overline{x} \cdot \overline{y}^T$).

3. Does the outer-product form help us to realize that AA^T is symmetric? Why or why not?