MAT225 Test 1 (Fall 2005): 1.1-2.3

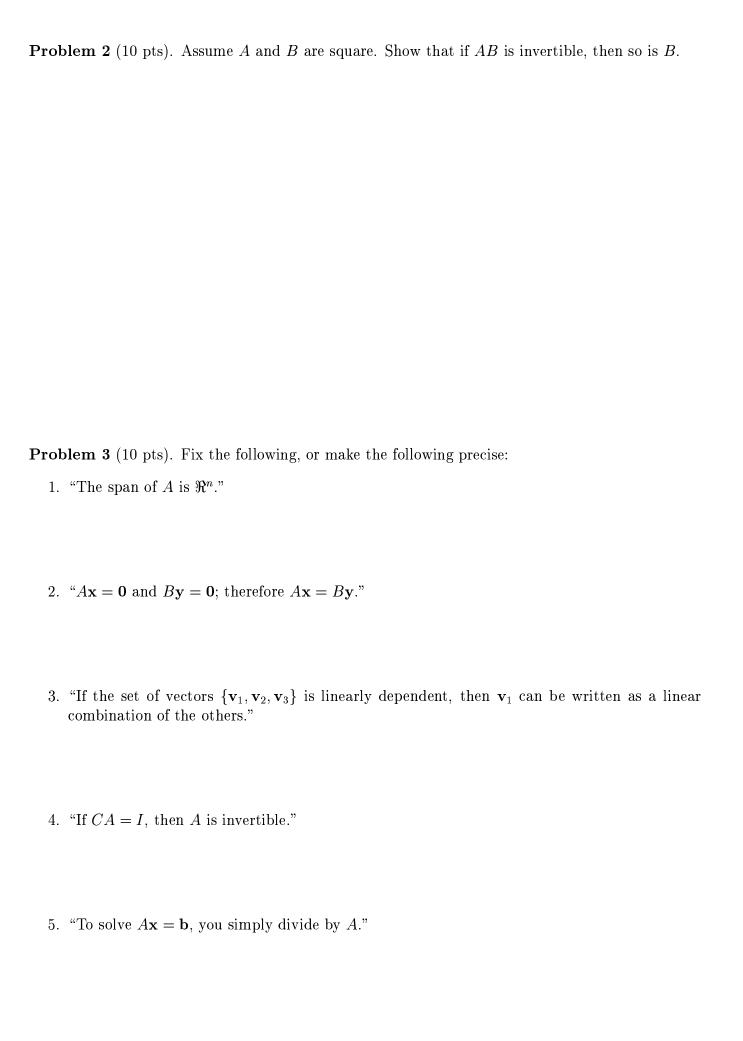**Name:**

Directions: Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1 (10 pts) Write the matrix

$$\begin{bmatrix} 2 & 1/2 & -2/3 \\ -1 & 3/4 & 0 \\ 4 & 0 & 0 \end{bmatrix}$$

in reduced row echelon form using partial pivoting, showing each step.

Problem 2 (10 pts). Assume A and B are square. Show that if AB is invertible, then so is B.

Problem 3 (10 pts). Fix the following, or make the following precise:

1. "The span of A is $\Re^n$."

2. "$A\mathbf{x} = \mathbf{0}$ and $B\mathbf{y} = \mathbf{0}$; therefore $A\mathbf{x} = B\mathbf{y}$."

3. "If the set of vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly dependent, then $\mathbf{v}_1$ can be written as a linear combination of the others."

4. "If $CA = I$, then A is invertible."

5. "To solve $A\mathbf{x} = \mathbf{b}$, you simply divide by A."

Problem 4 (10 pts). A is invertible. What can you say about

1. the dimensions of A?

2. The solutions of $A\mathbf{x} = \mathbf{0}$?

3. A^T?

4. the invertibility of CA?

Problem 5 (10 pts). Find the inverse of the matrix

$$A = \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}$$

by

1. the special formula for the inverse of a 2×2 matrix, and by

2. row reduction.

If you apply this matrix to a nose drawn in the plane, what happens to the nose? In other words, what does the linear transformation $T : \mathbf{x} \longrightarrow A\mathbf{x}$ do to the plane?

Problem 6 (10 pts). Write the matrices corresponding to the following linear transformations from $\Re^2$ to $\Re^2$:

1. A reflection about the y-axis.

2. A reflection about the line $y = -x$.

3. A scaling of the space by a factor of $2/3$.

4. A clockwise rotation of the space by an angle θ (you might try a couple of special cases).

Problem 7 (10 pts). Find examples that demonstrate the following (perhaps surprising) facts of matrix multiplication:

1. "$AB = 0$ does not imply that either $A = 0$ or $B = 0$."

2. "In general, $AB \neq BA$."

3. "$AB = 0$ and $BC = 0$ does not imply that $AB = BC$."

4. "$AB = AC$ does not imply that $B = C$."

Problem 8 (10 pts). Two power plants produce pollution, but in different ratios: one produces mercury and lead in the ratio 8 grams to 3 grams per ton of coal burned, and the other produces the same pollutants, but in the exactly opposite ratio of 3 to 8. Upon measuring the ratio of the pollutants in the environment about the two plants, the EPA determines that the ratio is 4 grams of mercury to 3 grams of lead.

Assume that there are no other sources of these two pollutants in the area:

1. If both plants were burning coal at the same rate, is it possible to have this ratio? Why or why not?

2. Find a relative rate of coal consumption that would lead to this ratio of mercury to lead.

Extra Credit (5 pts). Describe matrix multiplication in its outer product form, including some examples.