

MAT225 Test 1: 1.1-2.3

Name:

Directions: Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1 (10 pts) Write the matrix

$$\begin{bmatrix} -3 & 6.5 & -10 \\ 6 & -12 & 24 \\ 2 & -3 & 10 \end{bmatrix}$$

in reduced row echelon form using partial pivoting, showing each step.

Problem 2 (10 pts). Compute the inverse of

$$A = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$$

by row reduction, and use it to solve the system $A\mathbf{x} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

Problem 3 (10 pts). The columns of A are linearly independent. What can you say about

1. $A\mathbf{x} = \mathbf{0}$?

2. $A\mathbf{x} = \mathbf{b} \neq \mathbf{0}$?

3. The number of pivots of A ?

4. The invertibility of A ?

Problem 4 (10 pts). AB is invertible. What can you say about

1. B^{-1} ?

2. A^{-1} ?

3. BA ?

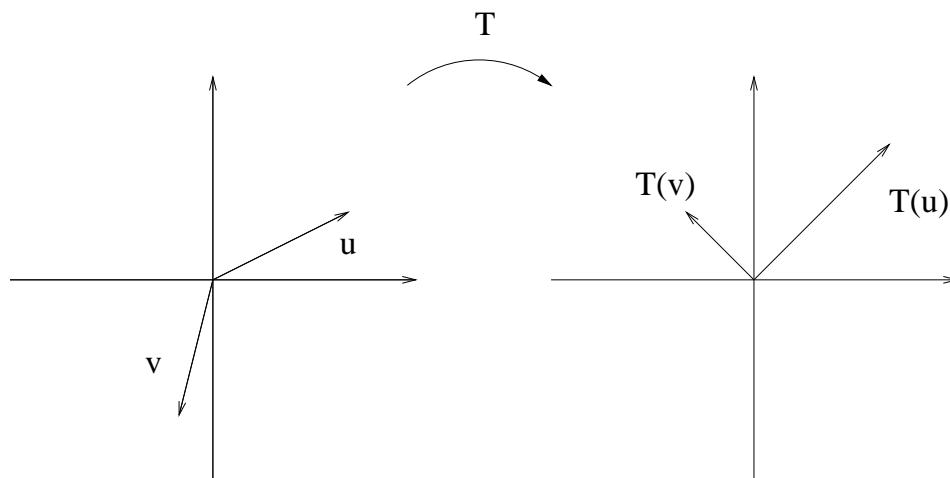
4. Suppose B is $n \times n$. How many pivots does B have?

Problem 5 (10 pts). Solve the linear systems

$$\begin{array}{l} 7x + 3.10y = .45 \\ 14x + 6.19y = .89 \end{array} \quad \text{and} \quad \begin{array}{l} 7x + 3.10y = .44 \\ 14x + 6.19y = .90 \end{array}$$

Comment.

Problem 6 (10 pts). Under the linear transformation T of \mathbb{R}^2 to \mathbb{R}^2 , the vectors \mathbf{u} and \mathbf{v} have images given as indicated in the following figures: In the figure to the right, draw the images of



$T(\mathbf{u} + \mathbf{v})$ and $T(\mathbf{u} - \mathbf{v})$. If $\mathbf{u} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} -1/2 \\ -2 \end{bmatrix}$, draw the image of $\begin{bmatrix} 5.5 \\ 8 \end{bmatrix}$.

Problem 7 (10 pts). Mark the following true (“T”) or false (“F”). If false, indicate how to fix the statement to make it true.

1. () If the columns of $A_{m \times n}$ span \mathbb{R}^m , $A\mathbf{x} = \mathbf{0}$ has a unique solution.
2. () If the condition number of A is infinite, its determinant is zero.
3. () A and A^T have the same number of pivot positions.
4. () If a linear system has more variables than equations, then it is over-determined.
5. () Geometrically, a system of three equations in three unknowns is equivalent to a question concerning the intersection of three planes.

Problem 8 (10 pts). Write the solution set (in parametric form) of the system $A\mathbf{x} = \mathbf{b}$, where

$$A = \begin{bmatrix} 5 & 5 & 0 & 0 \\ -3 & -10 & -7 & 0 \\ 8 & 13 & 5 & -1 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 5 \\ -17 \\ 17 \end{bmatrix}$$

Describe what the solution represents geometrically.

Extra Credit (5 pts). Consider the elementary matrix E corresponding to a row replacement. What is its inverse?