NKU CSC 480 Spring 2006 Assignment 2 Solutions

1a. $R_x(30°) = \begin{bmatrix} \sqrt{3}/2 & 0 & 1/2 & 0 \\ 0 & 1 & 0 & 0 \\ -1/2 & 0 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0866 & 0 & 0.5 & 0 \\ 0 & 1 & 0 & 0 \\ -0.5 & 0 & 0.866 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. Inverse: $R_x(30°)^T = \begin{bmatrix} \sqrt{3}/2 & 0 & -1/2 & 0 \\ 0 & 1 & 0 & 0 \\ 1/2 & 0 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

1b. $R = R_z(45°)^T\, R_x(60°) R_z(45°)$

$$= \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/2 & -\sqrt{3}/2 & 0 \\ 0 & \sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3/4 & -1/4 & -\sqrt{3/2}/2 & 0 \\ -1/4 & 3/4 & -\sqrt{3/2}/2 & 0 \\ \sqrt{3/2}/2 & \sqrt{3/2}/2 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.75 & -0.25 & -0.612 & 0 \\ -0.25 & 0.75 & -0.612 & 0 \\ 0.612 & 0.612 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Inverse: $R = R_z(45°)^T\, R_x(60°)^T R_z(45°) = \begin{bmatrix} 3/4 & -1/4 & \sqrt{3/2}/2 & 0 \\ -1/4 & 3/4 & \sqrt{3/2}/2 & 0 \\ -\sqrt{3/2}/2 & -\sqrt{3/2}/2 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

1c. M = (reflect thru y=0 then translate y by -3)(translate y by +3) = $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 6 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

Inverse: Same! (You undo a reflection by reflecting again!.)

1d. S= (scale by 10 and translate by d)(translate by –d) = $\begin{bmatrix} 10 & 0 & 0 & 2 \\ 0 & 10 & 0 & -1 \\ 0 & 0 & 10 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 & -18 \\ 0 & 10 & 0 & 9 \\ 0 & 0 & 10 & -9 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

Inverse = (scale by 1/10 and translate by d)(translate by –d) = $\begin{bmatrix} 1/10 & 0 & 0 & 2 \\ 0 & 1/10 & 0 & -1 \\ 0 & 0 & 1/10 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/10 & 0 & 0 & 9/5 \\ 0 & 1/10 & 0 & -9/10 \\ 0 & 0 & 1/10 & 9/10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

2. Midpoint= (A+B+C)/3 = (x/3, y/3, z/3). Note order with right hand rule (A,B,C):
perpendicular to face: $\mathbf{e}_3 \times$ (B-O) = $[\,-y, x, 0\,]^T$. Normalize to [-y/r, x/r, 0] where $r = \sqrt{(x^2 + y^2)}$.

3. $R = T^{-1}F^TR_z(180°)FT$, where

T= (translate C back to A) = $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -z \\ 0 & 0 & 0 & 1 \end{bmatrix}$. So $T^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{bmatrix}$. F= (rotate B–A to $\mathbf{e}_3$)

So F^T = (rotate $\mathbf{e}_3$ to B–A). So column 3 of F^T is just B-A. To get an orthogonal F^T, pick column 2 as a unit vector orthogonal to column 3: can use ($\mathbf{e}_3 \times$ B–A)^. Then pick column 1 as column 2 $\times$ column 3. This yields:

$F^T = \dfrac{1}{rc}\begin{bmatrix} -xz & yz & -c^2 \\ -yr & xr & 0 \\ xc & yc & -rz \end{bmatrix}$ where $c = \sqrt{(x^2 + y^2)}$ and $r = \sqrt{(x^2 + y^2 + z^2)}$. R maps the origin to $(2xz^2/r^2,\ 2yz^2/r^2,\ 2c^2z/r^2)$.

4. P = (translate z+=5) (project onto z=0 with eye at d=7)(translate z–=5)

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1/7 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 5/7 & -10/7 \\ 0 & 0 & -1/7 & 2/7 \end{bmatrix}. \quad P\begin{bmatrix} 3 \\ 3 \\ -10 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ -60/7 \\ 12/7 \end{bmatrix} \to (7/4, 7/4, -5).$$